Durchschnittswertindizes **Aus:**

6.4. Price indices and unit value indices, foreign trade and wages indices

For the k-th subcollection of related (homogeneous) commodities the so called unit values (a kind of average prices), $\tilde{p}_{k0}$ and $\tilde{p}_{kt}$ are defined by

(6.4.1) $$\tilde{p}_{k0} = \frac{\sum p_{kj0}q_{kj0}}{\sum q_{kj0}} = \frac{\sum p_{kj0}q_{kj0}}{Q_{k0}} \text{ and } \tilde{p}_{kt} = \frac{\sum p_{kjt}q_{kjt}}{\sum q_{kjt}} = \frac{\sum p_{kjt}q_{kjt}}{Q_{kt}}$$

where the summation takes place over $j = 1, 2, \ldots, m_k < n$ goods being a subcollection of all n goods. Consider now K groups (k = 1, ..., K), each containing m_k commodities such that there are $n = \sum_{k=1}^{K} m_k$ commodities altogether. The value index formula on the basis of unit values (instead of prices) is given by

(6.4.2) $$V_{0t} = \frac{\sum_k \tilde{p}_{kt}Q_{kt}}{\sum_k \tilde{p}_{k0}Q_{k0}} = \frac{\sum_k^K \sum_j^{m_k} p_{kjt}q_{kjt}}{\sum_k^K \sum_j^{m_k} p_{kj0}q_{kj0}} = \frac{V_t}{V_0}$$

(j = 1, 2, ..., m_k commodities within the k-th group). A Laspeyres type price index based on unit values of groups of commodities, to be called PU_{0t}^L is given by

(6.4.3) $$PU_{0t}^L = \frac{\sum_k \tilde{p}_{kt}Q_{k0}}{\sum_k \tilde{p}_{k0}Q_{k0}} = \frac{\sum_k^K \left(\sum_j^{m_k} \frac{p_{kjt}q_{kjt}}{Q_{kt}} \right) Q_{k0}}{\sum_k^K \sum_j^{m_k} p_{kj0}q_{kj0}} = \frac{\sum_k V_{kt} \frac{Q_{k0}}{Q_{kt}}}{V_0}$$ and the corresponding

Paasche index by

(6.4.4) $$PU_{0t}^P = \frac{\sum_k \tilde{p}_{kt}Q_{kt}}{\sum_k \tilde{p}_{k0}Q_{kt}} = \frac{\sum_k^K \sum_j^{m_k} p_{kjt}q_{kjt}}{\sum_k^K \left(\sum_j^{m_k} \frac{p_{kj0}q_{kj0}}{Q_{k0}} \right) Q_{kt}} = \frac{V_t}{\sum_k V_{k0} \frac{Q_{kt}}{Q_{k0}}}$$

Note that PU_{0t}^L (and PU_{0t}^P), like PU_{0t}^{UD} will not necessarily meet the mean value condition. To see this we express PU_{0t}^L as weighted mean of price relatives as follows

(6.4.3a) $$PU_{0t}^L = \sum_k^K \left(\sum_j^{m_k} \frac{p_{kjt}}{p_{kj0}} p_{kj0}q_{kjt} \right) \frac{Q_{k0}}{Q_{kt}} \Big/ \sum_k^K \sum_j^{m_k} p_{kj0}q_{kj0},$$ where the denominator is $\sum_k^K \sum_j^{m_k} p_{kj0}q_{kj0} = V_0$ .Summation of the weights in equation 3a leads to

(6.4.5) $$S = \sum_k^K \left(\sum_j^{m_k} p_{kj0}q_{kjt} \right) \frac{Q_{k0}}{Q_{kt}} = \sum_k^K \left(\sum_j^{m_k} p_{kj0}q_{kjt} \frac{\sum_j^{m_k} q_{kj0}}{\sum_j^{m_k} q_{kjt}} \right)$$

and there is no reason why this sum should necessarily equal to V_0

$$V_0 == \sum_k^K \left(\sum_j^{m_k} p_{kj0} q_{kjt} \frac{q_{kj0}}{q_{kjt}} \right).$$

Hence the value of PU^L can be less than the smallest, or greater than the greatest individual price relative (and the same is true for PU^P). Moreover unit value indices, PU are affected by changes in the composition of quantities within the K subcollections (groups) and they can indicate a rise (decline) of prices although all prices remained constant[1] (i.e. they can violate identity), simply due to changes in quantities. That this can happen will be shown in **ex. 7.2.1**.

Unit value indices PU violate the mean value property, and they therefore do not satisfy proportionality (nor do they satisfy identity). A change in unit values can well result from structural changes in the quantities, such that PU in general does not reflect a pure price movement[2]. On the other hand: the more detailed the product groups are defined (the classification is broken down) and the more homogeneous therefore the groups are the closer unit value indices like PU will come to true price indices P (and QU to Q respectively).

Unit value indices PU can indicate a change, even though all prices remain constant if there is a shift from one variant of an item (both falling into the same k-th subgroup) to another:

As a rule PU will understate the rise of prices (as compared to a true price index P) if there is a tendency to buy (import) or sell (export) more and more the cheaper commodities (at the expense of the more expensive ones), i.e. if there is a change in the structure of the groups in favor to the cheaper commodities. Conversely PU will overstate the price movement when the structure changes in favor to the more expensive goods.

Consequently volume indices weighted with unit values, to be called QU will overstate (understate) a rise in quantities (of export or import respectively) as compared with a true quantitiy index when PU understates (overstates) the rise of prices. This can easily be seen along with the violation of identity as follows:

Demonstration

Assumue only two commodities comprising a commodity group with prices $p_{10} = p_{1t} = p$ and $p_{20} = p_{2t} = \lambda p$, such that prices of both goods in fact remain constant. Furthermore consider shares $\alpha_{10} = \alpha_0$ and $\alpha_{20} = 1 - \alpha_0$ at the total quantity $Q_0 = q_{10} + q_{20}$ (such that $\alpha_0 = q_{10}/Q_0$) in the base period 0, and the shares $\alpha_{1t} = \alpha_t$ and $\alpha_{2t} = 1 - \alpha_t$ at the total quantity Q_t in period t respectively. To be more concrete we may also assume $\alpha_0 = ½$ and $\lambda = 1.5$. The situation thus is

good	price in 0	quantity share in 0	price in t	quantity share in 0
1	p	$\alpha_0 = q_{10}/(q_{10} + q_{20}) = 1/2$	p	$\alpha_t = q_{1t}/(q_{1t} + q_{2t})$
2	$\lambda p = 1.5p$	$1 - \alpha_0 = 1/2$	$\lambda p = 1.5p$	$1 - \alpha_t$
unit value	$p[\alpha_0 + \lambda(1-\alpha_0)] = 1.25p$		$p[\alpha_t + \lambda(1-\alpha_t)]$	

The ratio of unit values of this k-th group then is given by

$R = (\alpha_t + \lambda(1-\alpha_t))/(\alpha_0 + \lambda(1-\alpha_0)) = 0.8 \cdot (\alpha_t + 1.5 \cdot (1-\alpha_t)) = 1.2 - 0.4\alpha_t$ giving the following results:

[1] In this case all price relatives are unity, and unless S = 1 the unit value index PU^L need not satisfy identity.

[2] Nor does QU represent a pure quantity movement.

α_t	0.2	0.4	0.5	0.6	0.8
R	1.12	1.04	1	0.96	0.88

$\alpha_t < 0.5 \rightarrow R > 1$ *	$\alpha_t > 0.5 \rightarrow R < 1$ *

* though all prices in t are the same as in 0!

As $\alpha_t > 0.5$ (move to the cheaper commodity 1) R declines from 1 to 0.8 (if $\alpha_t = 1$). In the same manner: as α_t goes down from 0.5 (R = 1) to 0 then R rises from 1 to 1.2, and thus indicating a change in the structure in favor of the more expensive commodity 2 (the share of which at period t is $1-\alpha_t$).

Figure 6.4.1: The structure of indices on the basis of unit values*

Indices on the basis of

unit values (UV)		prices	
UV price indices	UV quantity indices	true price indices	true quantity indices
summation over k = 1, ... ,K groups $\sum_k$		summation over n commodities $\sum_k \sum_j$	
formula of Laspeyres		formula of Laspeyres	
$PU_{0t}^L = \frac{\sum \tilde{p}_t Q_0}{\sum \tilde{p}_0 Q_0}$	$QU_{0t}^L = \frac{\sum Q_t \tilde{p}_0}{\sum Q_0 \tilde{p}_0}$	$P_{0t}^L = \frac{\sum\sum p_t q_0}{\sum\sum p_0 q_0}$	$Q_{0t}^L = \frac{\sum\sum q_t p_0}{\sum\sum q_0 p_0}$
formula of Paasche		formula of Paasche	
$PU_{0t}^P = \frac{\sum \tilde{p}_t Q_t}{\sum \tilde{p}_0 Q_t}$	$QU_{0t}^P = \frac{\sum Q_t \tilde{p}_t}{\sum Q_0 \tilde{p}_t}$	$P_{0t}^P = \frac{\sum\sum p_t q_t}{\sum\sum p_0 q_t}$	$Q_{0t}^P = \frac{\sum\sum q_t p_t}{\sum\sum q_0 p_t}$

* The universe of n commodities is partitioned into K groups (subcollections) of related commodities; the subscript k = 1, 2, ..., K denotes the number of the group and the subscript j the j-th comoditiy of the k-th group.

In PU unit values are weighted with quantities and in QU quantities weighted with unit values (instead of prices). Therefore the following indentities hold

$$V_{0t} = PU_{0t}^L QU_{0t}^P = PU_{0t}^P QU_{0t}^L = \frac{\sum p_t q_t}{\sum p_0 q_0}, \tag{6.4.6}$$

in the same manner as $V_{0t} = P_{0t}^L Q_{0t}^P = P_{0t}^P Q_{0t}^L$ by definition. Hence the same value decomposition as known for price- and quantity-indices holds also for unit value indices.

Equation 6 also explains why PU^P is also used as deflators (esp. in the case of external trade). The result of deflation using PU_{0t}^P (instead of P_{0t}^P) is QU_{0t}^L, however, instead of Q_{0t}^L. The expressions QU^L and QU^P are also known as "*volume indices*"[3].

The different behavior of PU in contrast to P and QU as opposed to Q is demonstrated also in a numerical example (see **ex 6.4.1**). The bias due to an uncontrolled change in the mix of commodities within a group, will of course disappear, the more groups will be distinguished.

[3] The term "volume index" is highly ambiguous, however. In practice the word is used for Q *and* for QU as well.

In the limiting case of each group containing only one commoditiy ($m_k = 1\ \forall k$, K = n) and therefore perfectly homogeneous groups the PU and QU indices and the P and Q indices will be identical of course.

Example 6.4.1

Consider a group of commodities denoted by A which is composed of two commodities 1 and 2 and a second group, B which contains only one commodity, called 3.

	p_o	p_t	q_o	q_t	p_t/p_o
1 (A)	8	10	5	q	1.25
2 (A)	4	7	5	10-q	1.75
3 (B)	6	9	5	5	1.5

The parameter q enables us to check various changes in the composition of group A and their effects on the unit value price index, and the unit value quantity index respectively. Obviously the following results are easily verified and fixed, i.e. not depending on the choice made with respect to q:

- the denominator of P_{0t}^L, PU_{0t}^L, QU_{0t}^L, and V_{0t} which is $\sum \tilde{p}_0 Q_0 = \sum\sum p_0 q_0 = 90$
- the denominator of PU_{0t}^P, which is $\sum \tilde{p}_0 Q_t = 6 \cdot 10 + 6 \cdot 5 = 90$
- the numerator of P_{0t}^L, $\sum p_t q_0$ (or $\sum\sum p_t q_0$) which amounts to (10+7+9)5 = 130
- the unit values at base time, $\tilde{p}_{Bt}$ and the quantities Q at both periods (Q_0, Q_t) as follows

	$\tilde{p}_0$	$\tilde{p}_t$	Q_0	Q_t
A	6		10	10
B	6	9	5	5

where in this box the only variable depending on q is $\tilde{p}_{At}$ (the shaded part of the box), which is given by $\tilde{p}_{At} = 7 + 0.3q$, a function linear in q, and reflective of the fact that a structural change in favor of the more expensive commodity 1 (1 is more expensive than 2 in 0 as well as in t) yields a higher value of $\tilde{p}_{At}$ with consequences für PU^L and QU^P.

		normal (true) index type	unit value index type
price	Lasp.	P_{0t}^L = 130/90 = 1.444	PU_{0t}^L = (115 + 3q)/90
price	Paasche	P_{0t}^P = (115 + 3q)/(70+4q)	$PU_{0t}^P = PU_{0t}^L$
quant.	Lasp.	Q_{0t}^L = (70+4q)/90	QU_{0t}^L = 1
quant.	Paasche	Q_{0t}^P = (115 + 3q)/130	QU_{0t}^P = 1

The reader should verify equation $V_{0t} = (115 + 3q)/90$. The identity of PU_{0t}^L and PU_{0t}^P (as well as of QU_{0t}^L and QU_{0t}^P) is only accidential because we assumed $Q_0 = Q_t = 15$ and $q(B)_0 = q(B)_t = 5$ such that $QU_{0t}^L = QU_{0t}^P = 1$. Hence in this example we only have a change in *structure*, but not in the total *amount* consumed. Our aim was to work out the influence of the structure on the results taken in isolation. It is now easy to try out some values of q

1) q = 2 such that the consumption of that particular commodity which experienced the *lower rise of price* (price relative 1.25 instead of 1.75) and which was relatively more

expensive at base period, that is commodity 1 is *reduced* relative to the base period (reduced from 5 to 2). This yields

prices	$PU^L_{0t} = 121/90 = 1.3444 < P^L_{0t} = 1.4444 < P^P_{0t} = 121/78 = 1.5513$
quantities	$QU^L_{0t} = 1 > Q^P_{0t} = 0.9308 > Q^L_{0t} = 0.8667$

2) q = 8 that is a change in favor of (a rise in consumption of) that particular commodity which experienced the lower rise of price (which is the reason for now P^P being less than P^L instead of $P^P > P^L$ as above) and which was relatively more expensive at base period ($p_{10}=8 > p_{20}=4$). Thus

prices	$PU^L_{0t} = 1.5444 > P^L_{0t} = > 1.4444 > P^P_{0t} = 1.3627$*
quantities	$QU^L_{0t} =1 < Q^P_{0t} = 1.0692 < Q^L_{0t} = 1.1333$

* Note that P^L is not necessarily an upper bound

As a consequence of $QU^L_{0t} = QU^P_{0t} = 1$ we get $PU^L_{0t} = PU^P_{0t} = V_{0t}$ in all these cases. Note that P^L_{0t} is the same in the case of q = 2 and 8 because this index is not affected by changes in the quantites ($q_{it} \neq q_{i0}$) and substitutions within the groups of goods.

The result is fully in line with a general relationship already derived from some considerations set out above:

Whenever the *structure* of quantities within a group of commodities *changes in favor* of relatively[4] *less expensive* commodities we get PU < P and QU > Q. Conversely a change to more expensive commodities leads to PU > P and QU < Q.

Hence a structural change *within* the groups of commodities results in an understating of unit value *price* indices PU (as compared with true price indices P) and overstating of unit value *quantity* indices QU (as compared with a true quantity index Q) and vice versa. ♦

Example 6.4.2

The following modification of ex. 6.4.1 will provide an illustration of the possibility that PU^L is not necessarily satisfying the mean value condition. Again the first two commodities are grouped together to the group A whilst the third commodity forms a single-commodity group, B on its own:

	p_0	p_t	q_0	q_t	p_t/p_0
1 (A)	55	60	3	6	1.0909
2 (A)	4	4.8	10	5	1.2
3 (B)	6	5	9	5	5/6 = 0.833

A price index now should take a value within the range between 0.833 and 1.2.

We get $PU^L = 1.926$ ($P^L= 1.054$) which is well beyond the upper boundary of the span of individual price relatives (1.926 > 1.2).

See download of sec. 6.4 for more details.

[4] compared with other commodities in the same group at base period.