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Problems with Chain Indices (III)

Quarterly National Accounts (QNA) and Annual National Accounts (ANA)

Course delivered at the European Central Bank Frankfurt

7. Chainlinking in QNA

7. Chainlinking in QNA

7.1 (1) Why special chain-linking methods?

1. Chain indices as deflators in QNA **as a consequence of** move to chain indices in the deflation methodology of ANA in the **SNA 1993**

2. Difficult problems with chaining in QNA in particular because:

   - Need for **consistency between QNA and ANA**: annual sum of quarterly aggregates should not differ from ANA results
   "quarterly chain may move counter to the annual one" (Kuhnert, Eurostat)

   - "**Drift**, occurring with cyclical price and quantity movements, is more problematic as these cycles are more common in QNA (seasonality!)" → price weights of the **previous year rather than** of the **previous quarter**" (Kuhnert)

   → theory is more difficult: double indication (y,q) $I_{1,1}$ $I_{1,2}$ $I_{1.3}$ $I_{1,4}$ $I_{2,1}$ $I_{2,2}$ $I_{2,3}$ $I_{2,4}$ not all elements are "linked" together, for example only $I_{2,4} = L_1 * I_{1,4}$ and $I_{3,4} = L_2 * I_{2,4}$

   - unlike the situation of annual indices there is a choice among *different "linking techniques*": annual overlap (**AO**), quarterly overlap (**QO**), over the year (**OY**)

3. Compared to traditional "constant prices" - volume indicators the **computational burden** of a permanent update of the price base is heavier (some re-valuations necessary)

7.1 (2) Why special chain-linking methods? (part 2)

4. Consequences of different **choices of index formulas** may be less pronounced (Fisher [smaller drift?] may have less formal advantages over Laspeyres)

5. **Seasonal adjustment***: changes in the price-weight-base of volumes (e.g. between Q4 in y and Q1 in y+1 may be seen (mistaken) as seasonal pattern; should seasonal adjusted (SA) or non-SA figures be chain-linked?

   A problem is in which order the following operations should be carried out:

   Chaining (C), seasonal adjustment (A), benchmarking (B): C – A – B ?

6. Experience shows that difference between methods might be negligibly small; (unless there are significant substitution processes) "no method is the uniformly superior method" (Handbook on Price and Volume Measurement)

7. While turning points seem to be robust over different chain-linking techniques, seasonal and working day adjustment and outlier detection can be affected.*

8. Benchmarking (QNA/ANA discrepancy) may interfere with outlier detection and business cycle analysis* and also seasonal adjustment*

* see Scheiblecker (2007) for 7 + 8

* more in part IV

1. Dimensions of comparability

		period (e. g. quarter)	
		same	different
year	same		**D1** between successive periods of one year (quarter-on-quarter) **(y, q) → (y, q-1)**
	different	**D2** between a period of the current year and the same period in the previous year **(y, q) → (y-1, q)**	**D3** **(y, q) → (y ± a, q ± b)** in particular between a fourth quarter **(y, q = 4)** and the first quarter of the next year **(y+1, q=1)**

It is impossible to ensure consistent comparisons in all three dimensions

7.1 (4) Overview of methods; evaluation criteria: comparisons, types of linking

quarterly indices

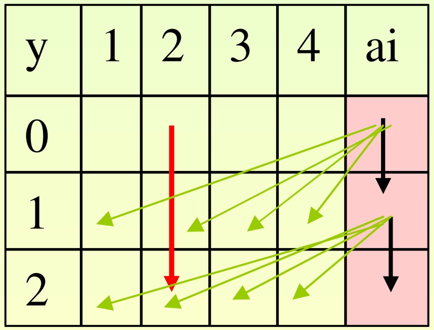
y	q = 1	q = 2	q = 3	q = 4	annual ind.
0	I_{01}	I_{02}	I_{03}	I_{04}	I_0
1	I_{11}	I_{12}	I_{13}	I_{14}	I_1
2	I_{21}	I_{22}	I_{23}	I_{24}	I_2

D1 ⟷ D2 ↕ D3 ↗

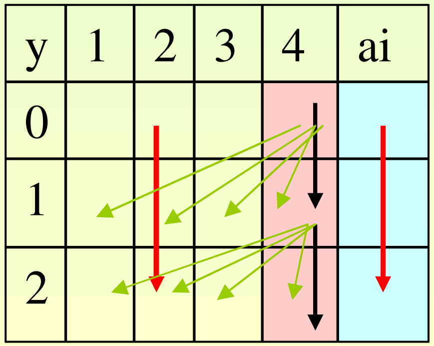
Consistent comparisons in all directions

1. pure quantity comparison (same price "weights")
2. no breaks caused by the method

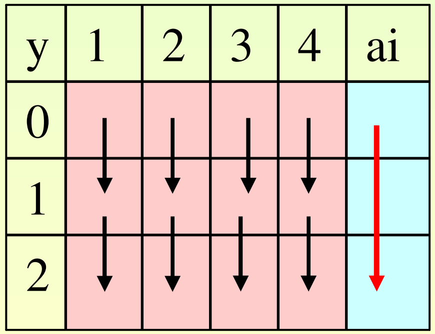
Annual **o**verlap



one **q**uarter **o**verlap



over the **y**ear (OY)



no linking in this direction

fields gained by multiplication

average of quarterly indices

series of indices all derived from multiplication (chainlinking)
Indices are forming a chain in this direction ⟶

2. Aggregation (requirements of consistent aggregation)

Aggregation

AC: over commodities (subaggregates)

AT: over intervals in time (time aggregation)

AC1: additivity of volumes

AC2: decomposition of growth rates how a GDP component contributes to total GDP growth

AT1: multiperiod identity (path dependence)

AT2: comparing periods of different length: consistency **between** cumulated **QNA and** direct **ANA**

All chaining procedures have poor aggregation properties!

3. Implementation (ease of compilation, data requirements)

e.g. QO and OY require calculation of $\overline{V}_{y-1,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y-1,q}$

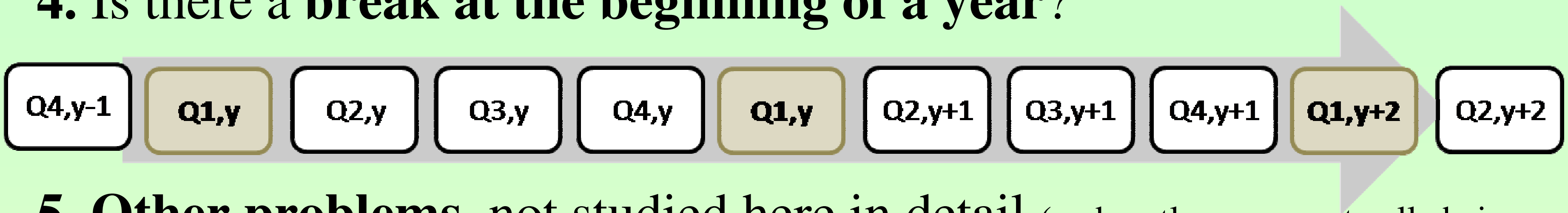
in addition to $\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}$

need to re-value quarters at prices of the current year

Notation V bar will be introduced later (slides 16 ff)

7.1. (6) Overview of methods Other evaluation criteria

4. Is there a **break at the beginning of a year**?



5. Other problems, not studied here in detail (and partly common to all chain-index problems)

1. decomposition of **growth rates** into "contributions" of certain goods or sub-aggregates

   *AO-method: growth rates of the total aggregates y,1 → y,2 → y,3 etc. can be consistently compared as they depend solely on volume changes (the same prices), yet when decomposed into "contributions" **weights** of the components are **not constant** (and depend on quantities)*
2. effects on (cumulated) aggregates like **fixed assets** (gross and net), accumulated capital consumption and the use of the perpetual inventory method (**PIM**)
3. reflection of the seasonal pattern and effect of various **seasonal adjustment** methods when applied to chained QNA data using different linking methods
4. effects of non-additivity on **econometric models** (definitional equations, sign of balancing items)

	Annual overlap (AO)	Quarterly overlap (QO)	over the year (OY)
Comparisons **D1 (y, q) → (y, q-1)**			
D2 (y, q) → (y-1, q)			
D3 (y, 4) → (y+1, 1)			
AC additivity and decomposition of growth rates			
AT consistency of ANA/QNA time aggregation			
Ease of computation (compilation)			

it is common to speak of "bias" if comparisons cannot consistently be made or AC AT etc, are not met*

final judgement of the three methods following this scheme see **section 7.6**

* strictly speaking "bias" applies to a sampling problem

$$E(\overline{x}) = \mu$$

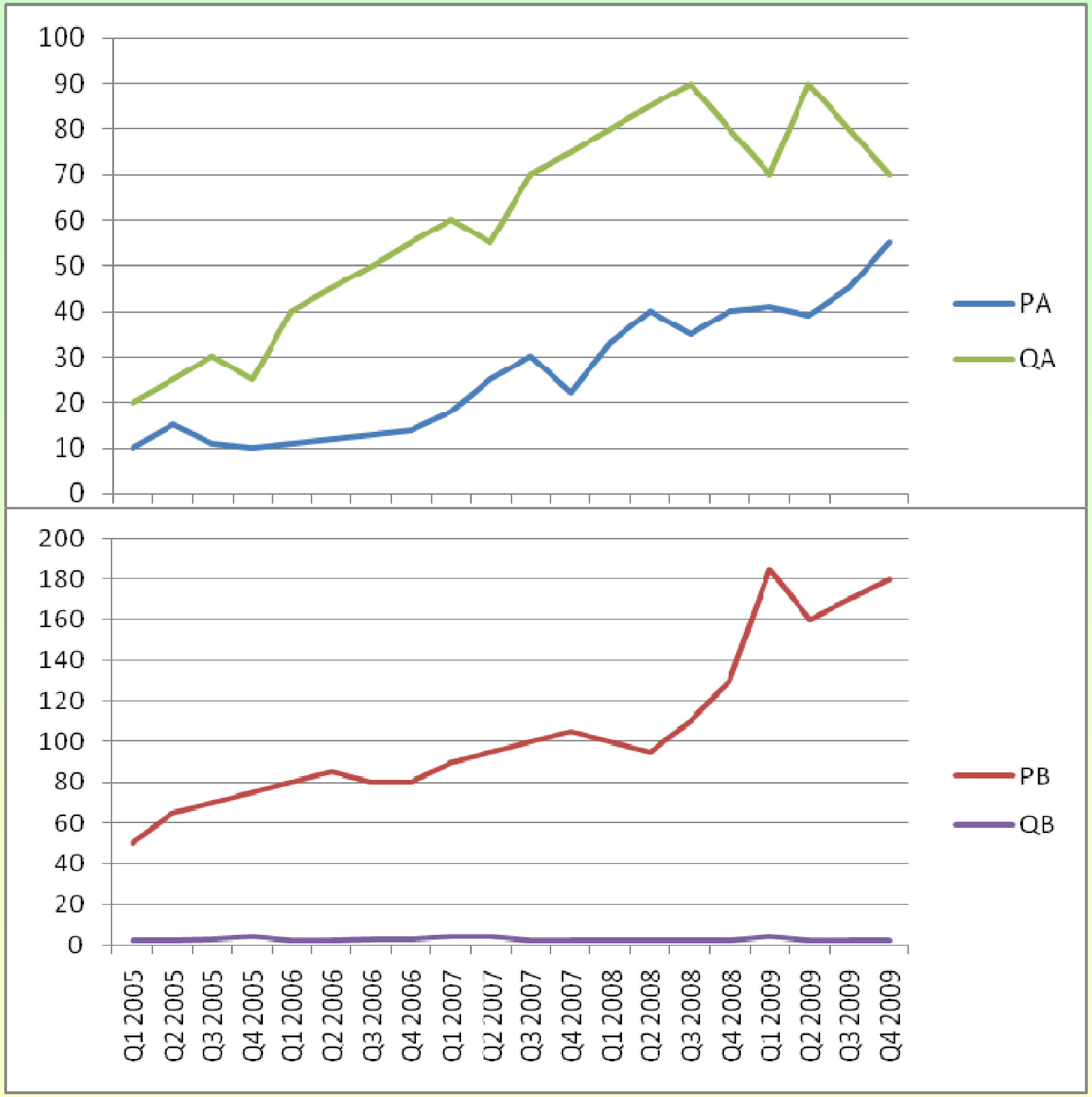
7.1 (8) Numerical example in the next section (1): assumptions

- It is difficult to understand the three methods AO, QO, and OY without resorting to a mostly somewhat laborious numerical example. Formulas in many papers or presentations are wrong or at least not fully transparent.
- Formulas are demonstrated using a numerical example (the numerical example of the IMF paper will also be presented): The fictitious data **average*** annual **prices and quantities** in 2005 to 2008 are as follows**:

	2005	2006	2007	2008	2009
average price of good A average price of good B	**11.55** **67.27**	**12.63** **81.00**	**23.87** **95.83**	**36.99** **108.75**	**44.61** **176.00**
index (2005 = 100) A B	100 100	109.35 120.42	206.67 142.45	320.26 161.66	386.23 261.62
av. quantity of good A av. quantity of good B	**25** **2.75**	**47.5** **2.5**	**65** **3**	**83.75** **2**	**77.5** **2.5**
index (2005 = 100) A B	100 100	190 90.9	260 109.1	335 72.7	310 90.9

* unweighted average of the four quarters of the year ** assumptions different from IMF-example

7.1 (9) Numerical example in the next section (1): Prices and quantities



in contrast: IMF example → **7.4.4**

other possible numerical examples (Kuhnert):

1. substitution effect
   - strong • weak
2. trend • yes • no
3. cycle • yes • no

7.1 (10) Numerical example in the next section (3): two types of volumes

- The fictitious data for 2005 to 2008 are such that volumes at constant (and average) prices of the base year 2005 and volumes at (average) prices of the previous year (and thus also the implicit price indices) differ a lot

	2005	2006	2007	2008	2009
(1) value (w) current prices	473.75	802.50	1838.75	3315.00	3897.50
(2) vol. const. 2005 prices	473.75	716.81	952.56	1101.86	1063.31
(3) volume at y-1 prices*	473.75	716.81	1064.05	2190.39	3138.22
implicit price index (1)/(2)**	100	111.95	193.03	300.85	366.54
implicit price index (1)/(3) **	100	111.95	172.81	151.34	124.19

It is legitimate to compare the two volumes (row 2 and 3) and form indices 2005 = 100 as done by deriving the implicit price indices

* multiplying links like 716.81/473.75 = **1.5131** and 952.56/716.81= **1.329** etc amounts to the same index 1.5131*1.329 = **2.0107** etc. (716.81 cancels out)

** rows 1 – 3 transformed into indices

7.1 (11) Numerical example in the next section (4): two types of volumes

Volumes at (average) **y-1 prices** are the **basis of all three methods** (AO, QO, and OY).

$$\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}$$

They seem to imply **a significantly higher growth** and lower inflation rate than **volumes at constant prices** of a fixed base period (e.g. 2005)

indices on the basis of row	2005	2006	2007	2008	2009
(1) value (w) current prices	100	169.39	388.13	699.73	822.69
(2) vol. const. 2005 prices	100	151.31	201.07	232.58	224.45
(3) volume at y-1 prices	100	151.31	224.60	462.35	662.42

However, it turns out that the final results (after chaining) generated by AO, QO, and OY are not very different from the traditional method using constant 2005 prices.

The reason is that **volumes at y-1 prices are not simply related to the base period value** - like volumes at constant prices of 2005 - but to other terms (see **7.2.6**) and thereafter chain-linked

7.2 (1) Steps common to all three methods: fundamental definitions and formulas

1. **General** principles of **volume definition** (price weights in volumes)

- the same prices for all quarters of the year as **annual deflator** (not prices of the previous quarter)
- quantity *weighted* average annual prices (= **unit values**) rather than unweighted arithmetic mean of quarterly prices (otherwise eq. → **6** would not hold)
- only **annual** chaining using unit value annual deflators of the **preceding year** (not of some constant base year) § 9.7-8, § 9.13-15*

2. Value and unit value

$$\bar{p}_{iy} = \frac{\sum_q p_{iyq} q_{iyq}}{\sum_q q_{iyq}} = \frac{W_{iy}}{Q_{iy}}$$

$$W_y = \sum_q W_{yq} = \sum_q \sum_i p_{iyq} q_{iyq}$$

p_{iyq} or q_{iyq}

i= good, commodity

y = year

q = quarter
q = 1,…,4

if applicable

* Paragraphs refer to the QNA Manual (of the IMF)

3. Various concepts of "volume" (at average prices of y-1) for quarters

$$V_{y,y\text{-}1,q} = V_{\text{quantities, prices, quarter of y}}$$ ← if applicable

prices	quarter-specific price	annual average price (unit value)
of y	(2) $W_{yq} = \sum_i p_{iyq} q_{iyq} = V_{y,y,q}$	(4) $\bar{V}_{y,y,q} = \sum_i \bar{p}_{i,y} q_{iy,q}$
of y-1	(3) $V_{y,y-1,q} = \sum_i p_{i,y-1,q} q_{i,y,q}$	(5) $\bar{V}_{y,y-1,q} = \sum_i \bar{p}_{i,y-1} q_{i,y,q}$

(4) is used as special case y = 0 for the start in all methods (4a) or as (4b) in the **OY** method (and esp. for q = 4 in the **QO** method)

$$\bar{V}_{0,0,q} = \sum_i \bar{p}_{i,0,q} q_{i,0,q}$$

$$\bar{V}_{y-1,y-1,q} = \sum_i \bar{p}_{i,y-1,q} q_{i,y-1,q}$$

(6) $$\bar{V}_{0,0,q} = \sum_i \bar{p}_{i0q} q_{i0q} = W_{0,q} = \sum_i p_{i0q} q_{i0q}$$

(3) Is the least relevant formula. In the formula handout it is shown, that (2), (4) and (5) indeed yield different results

7.2 (3) Steps common to all three methods: volumes (at average prices of y-1) 2005-2006

0 = 2005
1 = 2006

		commodity A			commodity B				
y	q	p_{Ayq}	q_{Ayq}	w_{Ayq}	p_{BVyq}	q_{Byq}	w_{Byq}	value W	volume V
0	1	10	**20**	200	50	**2**	100	300*	365.55*
	2	15	**25**	375	65	**2**	130	505**	423.30
	3	11	**30**	330	70	**3**	210	540	548.32
	4	10	**25**	250	75	**4**	300	550	557.84
sum/aver.		11.55	**100**	1155	67.27	**11**	740	473.75#	473.75
1	1	11	**40**	440	80	**2**	160	600	596.55**
	2	12	**45**	540	85	**2**	170	710	654.30
	3	13	**50**	650	80	**3**	240	890	779.32
	4	14	**55**	770	80	**3**	240	1010	837.07
sum/aver.		12.63	**190**	2400	81.00	**10**	810	802.50	716.81

$11.55 = \Sigma p_A q_A / \Sigma q_A = 1155/100$

$12.63 = 2400/190$

67.27 = 740/11

values = W
* = 100+200
** = 375+130

* = 20*11.55+2*67.27 **(4a)**
= 40*11.55 +2*67.27 **(5)

unweighted average = (300 + … + 550)/4

7.2 (4) Steps common to all three methods: volumes (at average prices of y-1) 2007-2009

		commodity A		commodity B		
y	q	p_{Ayq}	q_{Ayq}	p_{BVyq}	q_{Byq}	value W
2007	1	18	**60**	90	**4**	1440
	2	25	**55**	95	**4**	1755
	3	30	**70**	100	**2**	2300
	4	22	**75**	105	**2**	1860
2008	1	33	**80**	100	**2**	2840
	2	40	**85**	95	**2**	3590
	3	35	**90**	110	**2**	3370
	4	40	**80**	130	**2**	3460
2009	1	41	**70**	185	**4**	3610
	2	39	**90**	160	**2**	3830
	3	45	**80**	170	**2**	3940
	4	55	**70**	180	**2**	4210

This slide is simply for the years 2007 – 2009 the continuation of the preceding slide

some quantities are needed later (for demonstrations in section **7.5.1**)

← in particular 07,2 – 07.4 and 09.1 – 09.2

average annual prices*

year	A	B
2007	23.87	95.83
2008	36.99	108.75
2009	44.61	176.00

figures are rounded

* they are unit values (= quantity weighted average prices)

7.2 (5) Steps common to all three methods: values, volumes and links (2005 – 2007)

	value	vol. (05)*	link (06)	vol. (06)	link (07)	index
2005	300	365.55				77.16
	505	423.30				89.35
	540	548.32				115.74
	550	557.84				117.75
	473.75	473.75 **				100
2006	600	596.55				quarterly index
	710	654.30				
	890	779.32				
	1010	837.07				
	802.50	716.81**				annual index
2007	1440			1081.89		quarterly index
	1755			1018.74		
	2300			1046.21		
	1860			1109.37		
	1838.75			1064.05**		annual index

Eq. (4a) (→ vol. (05), 2005)

(5) (→ vol. (05), 2006)

Index continued using links for 06

using Ø prices of 2006 (preceding year) (5)

77.16 = (365.55/473.75)*100

89.35 = (423.30/473.75)*100

The three methods differ with respect to the definition and computation of the links

Index will be continued using the links for 07

* In prices of 2005   ** unweighted arithmetic mean

7.2 (6) Steps common to all three methods: values, volumes and links (2007 – 2009)

value	vol. (06)	link (07)	vol. (07)	link (08)	vol. (08)	link (09)	index
2007 1440 1755 2300 1860	1081.89 1018.74 1046.21 1109.37						
1838.75	1064.05						
2008 2840 3590 3370 3460			2100.90 2220.22 2339.55 2100.90				quarterly index
3315			2190.39				annual index
2009 3610 3830 3940 4210					3023.96 3546.16 3176.31 2806.46		quarterly index
3897.50					3138.22		annual index

Eq. (5)

$$\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}$$

7.2 (7) General approach: all methods

A quarter q of year y at average prices of the preceding year y- 1, that is

$$\overline{V}_{y,y-1,q} = \sum_i \overline{p}_{i,y-1} q_{i,y,q}$$

↑ numerator — denominator ↓

is related to

Annual overlap (AO)	a forth of the unweighted average of values of the preceding year y-1, that is to $W_{y-1}/4$ *
Quarterly overlap (QO)	the volume of q = 4 in y-1 at average prices of y-1 (eq. **(4b)** for all quarters)
Over the year (OY)	the same quarter of the preceding year y-1 (that is q, y-1) at *average* prices of the preceding year y-1 (eq. **(4b)** for q = 4)

For q = 4 both methods QO and OY yield the same result

* Therefore time consistency

7.3 Formulas for the indices

In this section we show - by means of **formulas** and a numerical **example** – how

- the index for y,q; y,q+1; … (sequence of **quarterly indices**) is derived
- **annual indices** (for y, y+1,..) are derived from linking and how they are related to the quarterly indices

in the case of the three techniques

7.3.1 annual overlap (AO)

7.3.2 quarterly overlap (QO)

7.3.3 over the year (OY)

In between (**7.4**) numerical results →

Later (**section 7.5**) it is shown which

- ♦ **comparisons** (in the three directions),
- ♦ **aggregations** (e.g. of QNA figures to directly gained ANA data) and
- ♦ **decompositions** of growth rates (into "contributions" of goods to growth)

can consistently be made

7.3.1 (1) Annual overlap (AO): fundamental formulas

	link	volume index
quarterly	(7) $L^{AO}_{(y-1)\to y,q} = \dfrac{\overline{V}_{y,y-1,q}}{W_{y-1}/4}$	(8) $I^{AO}_{y,q} = I^{AO}_{y-1} \cdot L^{AO}_{(y-1)\to y,q}$
annually	(9) $L^{AO}_{(y-1)\to y} = \dfrac{\sum_q L^{AO}_{(y-1)\to y,,q}}{4}$ or $L^{AO}_{(y-1)\to y} = \overline{V}_{y,y-1}/W_{y-1}$	(10) $I^{AO}_{y} = I^{AO}_{y-1} \cdot L^{AO}_{(y-1)\to y}$

aggregation of QNA and direct ANA are compatible

(9) $$L^{AO}_{(y-1)\to y} = \frac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q W_{y-1.q}} = \frac{\overline{V}_{y,y-1}}{W_{y-1}} = \frac{\frac{1}{4}\sum_q \overline{V}_{y,y-1,q}}{\frac{1}{4} W_{y-1}} = \frac{\sum_q L^{AO}_{(y-1)\to y,,q}}{4}$$

This formula proves that growth rate of annual index equals growth of accumulated QNA aggregates (= “time consistency”)

7.3.1 (2) Annual overlap (AO): in one single formula

Index I^{AO} for quarter q = 2 in year y = 4 expressed in one single formula

$$\left(\prod_{t=1}^{y-1}\frac{\sum_q \overline{V}_{t,t-1,q}}{\sum_q W_{t-1.q}}\right)\frac{\overline{V}_{y,y-1,q}}{W_{y-1.q}/4}=\left(\prod_{t=1}^{y-1}\frac{\sum_q \overline{V}_{t,t-1,q}}{W_{t-1}}\right)\frac{\overline{V}_{y,y-1,q}}{W_{y-1}/4}$$

$$=\left(\frac{\sum_q\sum_i \overline{p}_0 q_{1q}}{\sum_q\sum_i p_{0q}q_{0q}}\frac{\sum_q\sum_i \overline{p}_1 q_{2q}}{\sum_q\sum_i p_{1q}q_{1q}}\frac{\sum_q\sum_i \overline{p}_2 q_{3q}}{\sum_q\sum_i p_{2q}q_{2q}}\right)\frac{\sum_i \overline{p}_3 q_{4q=2}}{\sum_q\sum_i p_{3q}q_{3q}/4}$$

Year 4 and quarter q = 3

$$\left(\frac{\sum_q\sum_i \overline{p}_0 q_{1q}}{\sum_q\sum_i p_{0q}q_{0q}}\frac{\sum_q\sum_i \overline{p}_1 q_{2q}}{\sum_q\sum_i p_{1q}q_{1q}}\frac{\sum_q\sum_i \overline{p}_2 q_{3q}}{\sum_q\sum_i p_{2q}q_{2q}}\right)\frac{\sum_i \overline{p}_3 q_{4q=3}}{\sum_q\sum_i p_{3q}q_{3q}/4}$$

Growth factor year 4, q = 2 → y=4, q = 3

$$\frac{I^{AO}_{4,3}}{I^{AO}_{4,2}}=\frac{\sum_i \overline{p}_3 q_{4q=3}}{\sum_i \overline{p}_3 q_{4q=2}}$$

Same growth factors as QO method (except for q = 1)

7.3.1 (3) Annual overlap (AO) 2005 - 2007

value		vol. (05)*	link (06)	vol. (06)	link (07)	index
300 505 540 550	2005	365.55 423.30 548.32 557.84				**77.16** **89.35** **115.74** **117.75**
473.75		473.75 **a**				100
600 710 890 1010	2006	596.55 654.30 779.32 837.07	125.92 **b** 138.11 **c** 164.50 176.69	→ (8)	→	**125.92** **138.11** **164.50** **176.69**
802.50		716.81	151.30 **d**			**151.30**
1440 1755 2300 1860	2007	$W_{2006/4}$ = 802.5		1081.89 1018.74 1046.21 1109.37	134.82 **e** 126.95 **e** 130.37 138.24	**203.98 f** **192.07 f** **197.25** **209.16**
1838.75				1064.05	132.59	**200.62**

a = unweighted arithm. mean = $W_y/4$ (2)

b : 596.55/473.75 = 1.2592 (7)

c : 654.30/473.75 = 1.3811 (7)

d : 716.81/473.75 = 1.5130 or unweighted mean (9)

e : 1081.89/802.5 = 1.3482 and 1.2695 = 1081.74 /802.5 (7)

f : 151.3*1.3482 = 203.98
151.3*1.2695 = 192.07

1064.05/802.50

* In prices of 2005

Verify: the same quarter-on-quarter growth rates as in the case of QO

7.3.1 (4) Annual overlap (AO) 2007 - 2009

value	vol. (06)	link (07)	vol. (07)	link (08)	vol. (08)	link (09)	index
… (2007)	…						…
1860	1109.37						**209.16**
1838.75	1064.05						**200.62**
2840 (2008)			2100.90	114.26 **a**			**229.22**
3590			2220.22	120.75			**242.24 b**
3370			2339.55	127.24			**255.26**
3460			2100.90	114.26			**229.22**
3315			2190.39	119.12 **c**			**238.98 d**
3610 (2009)					3023.96	91.22 **e**	**218.00**
3830					3546.16	106.97	**255.65**
3940					3176.31	95.82	**228.99**
4210					2806.46	84.66	**202.32**
3897.50					3138.22	94.67	**226.24**

$W_{2007/4} = 1838.75$

a = 2100.9/1838.75 = 1.1426

c = 2190.39/1838.75= 1.1912

b = 200.62*1.2075

d = 200.62*1.1912

e = 3023.96/3315= 0.9122

7.3.2 (1) Quarterly overlap (QO): fundamental formulas

	link	volume index
quar-terly	(11)* $L^{QO}_{y-1,q=4\rightarrow y,q} = \dfrac{\bar{V}_{y,y-1,q}}{\bar{\bar{V}}_{y-1,y-1,q=4}}$	(12)* $I^{QO}_{y,q} = I^{QO}_{y-1,q=4} L^{QO}_{y-1,q=4\rightarrow y,q}$
annual-ly	(13) $L^{QO}_{y-1,q=4\rightarrow y} = \dfrac{\sum_q L^{QO}_{y-1,q=4\rightarrow y,q}}{4}$	(14) $I^{QO}_{y} = I^{QO}_{y-1,q=4} L^{QO}_{y-1,q=4\rightarrow y}$ (not I^{AO}_{y-1})

(11*) start (y = 1): $L^{QO}_{0,q=4\rightarrow 1,q} = \dfrac{\bar{V}_{1,0,q}}{\bar{\bar{V}}_{0,0,q=4}}$

(12a) starting with $I^{QO}_{0,q=4} = \dfrac{\bar{V}_{0,0,q}}{W_0/4}$

Note: not the annual indices but only y, q = 4 indices can be written as a "chain" (product)

The fact that $L^{QO}_{y-1,q=4\rightarrow y} = \dfrac{\frac{1}{4}\sum_q \bar{V}_{y,y-1,q}}{\bar{\bar{V}}_{y-1,y-1,q=4}} = \dfrac{\sum_q L^{OY}_{y-1,q=4\rightarrow y,q}}{4}$ should not be misunderstood as if aggregation of QNA and direct ANA were compatible (time consistent): see formula handout p. 7 and eq. 13b on the following slide

here an unweighted average like in (9) slide 23

7.3.2 (2) QO fundamental formulas: sequence of annual indices

$$I_{y,4}^{QO} = \frac{\overline{V}_{1,0,q=4}}{\frac{1}{4}W_0}\frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1,1,q=4}}\frac{\overline{V}_{3,2,q=4}}{\overline{V}_{2,2,q=4}}\cdots\frac{\overline{V}_{y,y-1,q=4}}{\overline{V}_{y-1,y-1,q=4}}$$

The sequence is given by (14a, 14b , …)

$$I_1^{QO} = I_{0,q=4}^{QO}L_{0,q=4\to 1}^{QO} = \frac{\overline{V}_{0,0,q=4}}{W_0/4}\frac{\frac{1}{4}\sum_q \overline{V}_{1,0,q}}{\overline{V}_{0,0,q=4}}$$

$$I_2^{QO} = I_{1,q=4}^{QO}L_{1,q=4\to 2}^{QO} = \frac{\overline{V}_{0,0,q=4}}{W_0/4}\frac{\overline{V}_{1,0,q=4}}{\overline{V}_{0,0,q=4}}\frac{\frac{1}{4}\sum_q \overline{V}_{2,1,q}}{\overline{V}_{1,1,q=4}}$$

$$I_3^{QO} = I_{2,q=4}^{QO}L_{2,q=4\to 3}^{QO} = \frac{\overline{V}_{0,0,q=4}}{W_0/4}\frac{\overline{V}_{1,0,q=4}}{\overline{V}_{0,0,q=4}}\frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1,1,q=4}}\frac{\frac{1}{4}\sum_q \overline{V}_{3,2,q}}{\overline{V}_{2,2,q=4}}$$

compare this with eq. 13 on slide 27

The growth factor of the annual volume is not a sum or unweighted average of quarterly growth factors but a **weighted** sum

(13b) annual growth:

weights in brackets

$$\frac{I_y^{QO}}{I_{y-1}^{QO}} = \frac{I_{y,q=1}^{QO}}{I_{y-1,q=1}^{QO}}\left(\frac{I_{y-1,q=1}^{QO}}{4\cdot I_{y-1}^{QO}}\right) + \ldots + \frac{I_{y,q=4}^{QO}}{I_{y-1,q=4}^{QO}}\left(\frac{I_{y-1,q=4}^{QO}}{4\cdot I_{y-1}^{QO}}\right)$$

no "time consistency"

7.3.2 (3) QO index in one formula

$$I^{QO}_{y,q} = \frac{\sum \bar{p}_0 q_{1;4}}{W_0/4} \cdot \left(\prod_{t=2}^{y-1} \frac{\sum \bar{p}_{t-1} q_{t;4}}{\sum \bar{p}_{t-1} q_{t-1;4}} \right) \cdot \frac{\sum \bar{p}_{y-1} q_{y;q}}{\sum \bar{p}_{y-1} q_{y-1;4}}$$

To better understand the formula we again assume y = 4 and q = 2 and use our notation

$$I^{QO}_{4;2} = \frac{\overline{V}_{1,0,q=4}}{W_0/4} \left(\frac{\overline{V}_{2,1,q=4}}{\overline{\overline{V}}_{1,1,q=4}} \cdot \frac{\overline{V}_{3,2,q=4}}{\overline{\overline{V}}_{2,2,q=4}} \right) \frac{\overline{V}_{4,3,q=2}}{\overline{\overline{V}}_{3,3,q=4}}$$

verified with our numerical example

$$I^{QO}_{4;2} = \frac{837.07}{473.75} \left(\frac{1109.37}{937.74} \cdot \frac{2100.90}{1981.57} \right) \frac{3546.16}{3176.31} = 2.4742$$

1.7669 →

2.0903 →

2.2162 →

for the numerators see slide 30 and 31

growth factor y=4,q=2 → y=4,q=3 $\dfrac{I^{QO}_{4,3}}{I^{QO}_{4,2}} = \dfrac{I^{AO}_{4,3}}{I^{AO}_{4,2}} = \dfrac{\sum_i \bar{p}_3 q_{4q=3}}{\sum_i \bar{p}_3 q_{4q=2}}$ cp slide 24

7.3.2 (4) Quarterly overlap (QO) 2005 - 2007

value		vol. (05)	link (06)	vol. (06)	link (07)	index
300	2005	365.55				**77.16**
505		423.30				**89.35**
540		548.32				**115.74**
550		**557.84**				**117.75**
473.75		473.75				100
600	2006	596.55	106.94 **a**			**125.92 c**
710		654.30	117.29			**138.11 d**
890		779.32	139.70 **b**			**164.50**
1010		837.07	150.06	***937.74*** **f**		**176.69**
802.50		716.81	128.50 **e**			151.30 **e**
1440	2007			1081.89	115.37 **g**	**203.85 h**
1755				1018.74	108.64	**191.95**
2300				1046.21	111.57	**197.13**
1860				1109.37	118.30	**209.03**
1838.75				1064.05	113.47 **i**	200.49 **i**

a: 596.55/557.84

b: 779.32/557.84

c: 117.75*1.0694

d: 117.75*1.1729

e: 716.81/557.84 = 1.285 and 151.3 = 117.75*1.285

f: quantities of 2006_IV at **average** prices of 2006 **(4b)** = 55*12.63+3*81

g: 1081.89/937.74

h: 176.69*1.1537

i: 1.133 = 1064.05/937.74 and 176.69*1.1347=200.49

7.3.2 (5) Quarterly overlap (AO) 2007 - 2009

	value	vol. (06)	link (07)	vol. (07)	link (08)	vol. (08)	link (09)	index
2007	1440	1081.89	115.37					**203.85**
	1755	1018.74	108.64					**191.95**
	2300	1046.21	111.57					**197.13**
	1860	1109.37	118.30	***1981.57***				**209.03**
	1838.75	1064.05	113.47					200.49
2008	2840			2100.90	106.02*			**221.62**
	3590			2220.22	112.04			**234.20****
	3370			2339.55	118.07			**246.79**
	3460			2100.90	106.02	***3176.31***		**221.62**
	3315			2190.39	110.54			231.06
2009	3610					3023.96	95.20	**210.99**
	3830					3546.16	111.64	**247.42**
	3940					3176.31	100.00	**221.62**
	4210					2806.46	88.36	**195.81**
	3897.50					3138.22	98.90	218.96

* = 2100.9/1981.57 **209.03*1.1204

7.3.3 (1) Over the year (OY): fundamental formulas

	link	volume index
quar-terly	(15)* $L^{OY}_{y-1,q\to y,q} = \frac{\overline{\overline{V}}_{y,y-1,q}}{\overline{\overline{V}}_{y-1,y-1,q}}$	(16) $I^{OY}_{y,q} = I^{OY}_{y-1,q} L^{OY}_{y-1,q\to y,q}$
annu ally	(17) $L^{OY}_{(y-1)\to y} = \frac{\sum_q \overline{V}_{y,y-1,q}}{\sum_q \overline{V}_{y-1,y-1,q}}$	(18) $I^{OY}_{y} = I^{OY}_{y-1} L^{OY}_{(y-1)\to y} \neq \frac{1}{4}\sum_q I^{OY}_{y,q}$

$$L^{OY}_{(y-1)\to y} = \frac{\sum_q I^{OY}_{y-1,q} L^{OY}_{(y-1),q\to y,q}}{\sum_q I^{OY}_{y-1,q}} \neq \frac{\sum_q L^{OY}_{(y-1),q\to y,q}}{4}$$

aggregation of QNA and direct ANA are **not** compatible (no time consistency)

OY virtually constructs **not one but rather four chains**, one for each quarter and the successive quarters are **not** linked together

* **compare (15) to (11)!**

7.3.3 (2) Over the year quarterly index in one formula

$$I_{y,q}^{OY} = \frac{\sum p_0 q_{0;q}}{W_0/4} \prod_{1=1}^{y} \frac{\sum \bar{p}_{t-1} q_{y;q}}{\sum \bar{p}_{s-1} q_{y-1;q}}$$

To verify assume again y = 4 and q = 2

$$I_{4,2}^{OY} = \frac{\sum p_0 q_{1;2}}{W_0/4} \frac{\sum \bar{p}_1 q_{2;2}}{\sum \bar{p}_1 q_{1;2}} \frac{\sum \bar{p}_2 q_{3;2}}{\sum \bar{p}_2 q_{2;2}} \frac{\sum \bar{p}_3 q_{4;2}}{\sum \bar{p}_3 q_{3;2}}$$

see the following slides for the figures of the numerical example

$$I_{4,2}^{OY} = \frac{654.30}{473.754} \frac{1018.74}{730.42} \frac{2220.22}{1695.93} \frac{3546.16}{3361.26} = 2.6605$$

1.3811 →

1.9263 →

2.5218 →

the terms $\sum \bar{p}_1 q_{1;2}$ $\sum \bar{p}_2 q_{2;2}$ $\sum \bar{p}_3 q_{3;2}$ have to be calculated especially for OY

7.3.3 (3) Over the year (OY) 2005 - 2007

value	vol. (05)	link (06)	vol. (06)	link (07)	index
300 505 540 550 2005 = 0	365.55 423.30 548.32 557.84				**77.16** **89.35** **115.74** **117.75**
473.75	473.75				100
600 710 890 1010 2006 = 1	596.55 654.30 779.32 837.07	163.19* 154.57** 142.13 150.06	667.26 730.42 874.58 *937.74*	see OY	**125.92** **138.11** **164.50** **176.69**
802.50	716.81	152.49	802.50		151.3
1440 1755 2300 1860 2007 = 2			1081.89 1018.74 1046.21 *1109.37*	***162.14*** ***139.47*** see OY	**204.17** **192.63**
1838.75			1064.05		

unweighted arithmetic mean (over the quarters), not 716.81/473.75 = 1.513055

These volumes are re-calculated for 2006 using **average** prices of **2006** (p_A = 12.63 and p_B = 81) 667.26 = **40***12.63 + **2***81 or 730.42 = **45***12.63 + **2***81

138.11 = 89.35*1.5457

164.50 = 115.75*1.4213

151.3: see the mean of 125.92, … 176.69 would be 158.055

192.63 =138.11*1.3947

162.14 = 1081.89/667.26

139.47 = 1018.74/730.42

* 596.55/365.55 = 1.6319 ** 654.30/423.30 = 1.5457

7.3.3 (4) Over the year (OY) 2007 - 2009

value		vol. (06)	link (07)	vol. (07)	link (08)	vol. (08)	link (09)	index
1440	2007 = 2	1081.89	***162.14***	1815.26				**204.17**
1755		1018.74	***139.47***	1695.93				**192.63**
2300		1046.21	119.62	1862.24				**196.78**
1860		1109.37	118.30	1981.57				**209.03**
1838.75		1064.05		1838.75				200.65 **a)**
2840	2008 = 3			2100.90	115.74 **b)**			**236.29 c)**
3590				2220.22	130.91			**252.18 d)**
3370				2339.55	125.63			**247.22**
3460				2100.90	106.02			**221.62**
3315				2190.39	119.58			239.33
3610	2009 = 4					3023.96	95.20	**224.96**
3830						3546.16	105.50	**<u>266.05</u>**
3940						3176.31	89.57	**221.43**
4210						2806.46	88.36	**195.81**
3897.50						3138.22	94.66	227.06

Ø prices in 2007
$p_A = 23.87$, $p_B = 95.83$
1815.26 = **60***23.87 +**4***95.95.83

2100.90/1815.26

1064.05 and 1838.75 are unweighted mean over the quarters (but rather ir-relevant)

unweighted mean over the quarterly indices

see slide 33

a) Unweighted mean over 204.17 + … + 209.03 (increase 32.6%)
not (1838.75/ 802.50)*100 = 229.13 (instead of 200.65)

c) 204.17*1.1574
d) 192.63*1.3091

7.4 Results of the numerical example: 1. quarterly indices and 2. annual indices according to the three methods and the traditional **constant prices** volume index (direct Laspeyres quantity index). We will look at **tables, graphs, correlations**

3. It is also considered what would happen if indices were **quarterly chained (re-weighted) rather than annually** (that is if the quarterly volumes would be multiplied [chained or "chain-linked"]

4. results of the numerical **example** of the **IMF manual** are also presented

7.5 More formulas: chained indices, and indices derived from them; formulas **for the comparisons D1, D2, and D3** and for the computation of **contribution to growth** (decomposition of growth rates)

7.6 Final discussion of **advantages and disadvantages** of the three methods as opposed to the traditional constant prices volume index

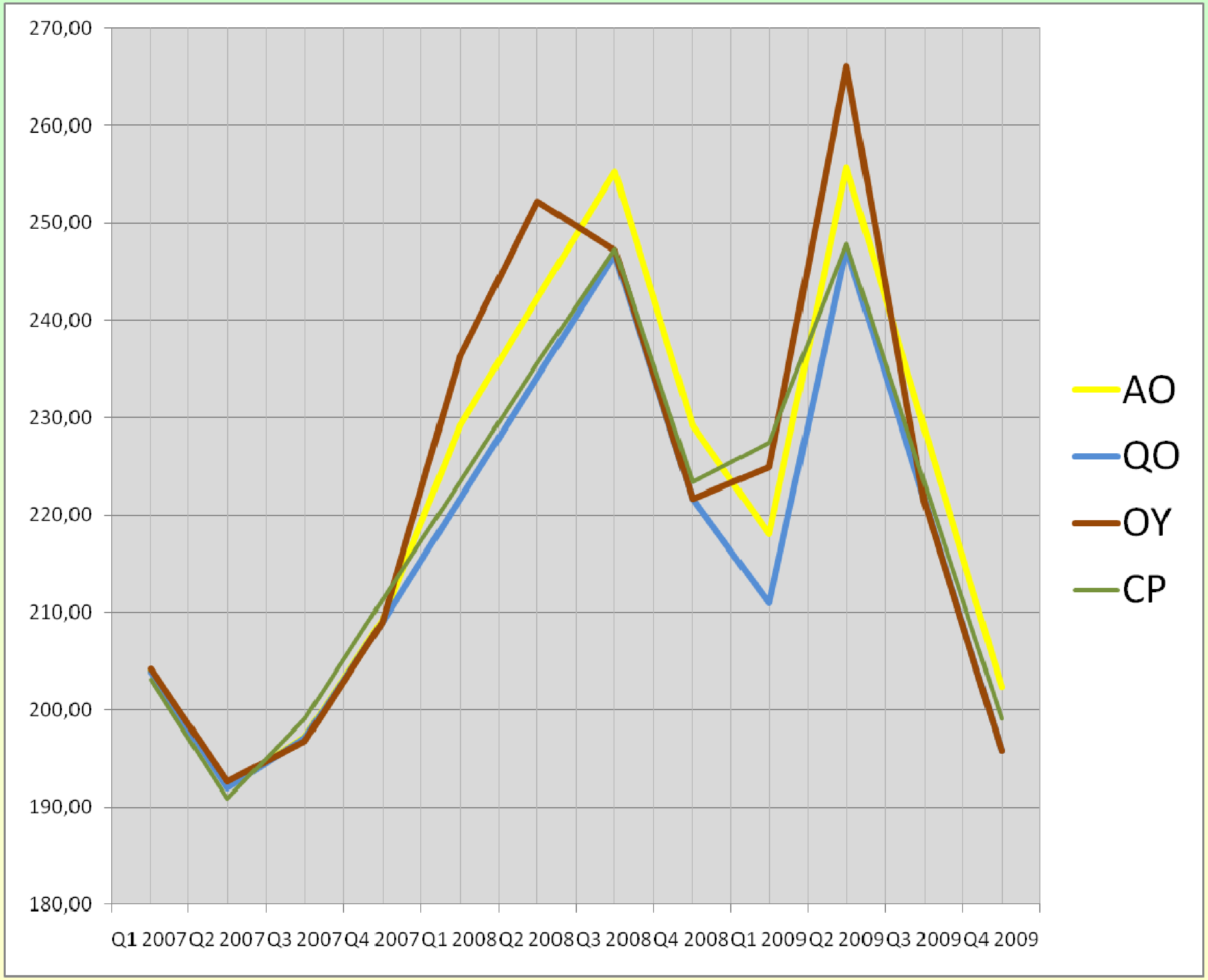
7.4.1 (1) Volumes based on constant prices, Synopsis of methods: quarterly indices

y	volumes*	index*	volumes**	index**	AO	QO	OY
2006	596.54	125.92		125.92	125.92	125.92	125.92
	654.29	138.11		138.11	138.11	138.11	138.11
	779.32	164.50		164.50	164.50	164.50	164.50
	837.07	176.69		176.69	176.69	176.69	176.69
2007	962.09	203.08	1081.89	228.37	203.98	203.85	204.17
	904.34	190.89	1018.74	215.04	192.07	191.95	192.63
	943.05	199.06	1046.21	220.84	197.25	197.13	196.78
	1000.79	211.25	1109.37	234.17	209.16	209.03 ↔	209.03
2008	1058.55	223.44	2100.90	443.46	229.22	221.62	236.29
	1116.30	235.63	2220.22	468.65	242.24	234.20	252.18
	1174.05	247.25	2339.55	494.84	255.26	246.79	247.22
	1058.55	223.44	2100.90	443.46	229.22	221.62 ↔	221.62
2009	1077.50	227.46	3023.96	638.30	218.00	210.99	224.96
	1174.05	247.82	3546.16	748.53	255.65	247.42	266.05
	1058.55	223.44	3176.31	670.46	228.99	221.62	221.43
	943.05	199.06	2806.46	592.39	202.32	195.81 ↔	195.81

* at constant average prices of 2005

** at average prices of the preceding year (much higher then at prices of 2005; see also slides 13/14)

7.4.1 (2) Graph of the quarterly indices



The results of the three methods are quite similar

constant prices (CP) volumes seem to be the least volatile indices →

7.4.1 (3) Results of the three methods (quarterly indices 2007 – 2009)

correlations (between indices)

	AO	QO	OY
QO	0,99325	1	
OY	0,95946	0,95795	1
CD	0,97731	0,97061	0,95930

other descriptive statistics

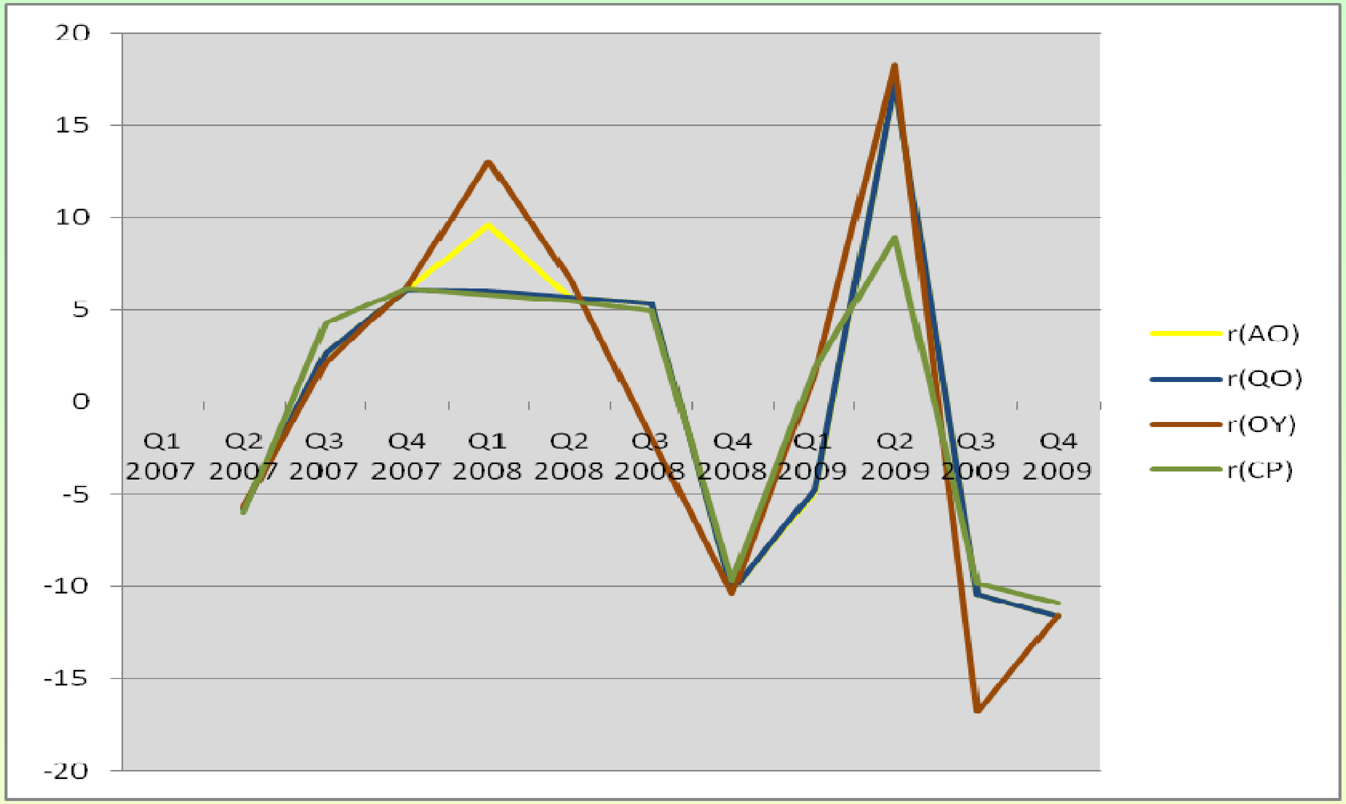
	AO	QO	OY	CP
SD	55,16	52,28	55,89	18,91
AM	183,43	180,36	183,67	219,36
CV	0,3007	0,2898	0,3043	0,0862
AD	46,22	43,77	46,41	15,54

This confirms: CP is the least volatile

SD = standard deviation; AM = arithmetic mean,
CV = coefficient of variation; AM = mean absolute deviation

7.4.1 (4) The three methods (quarterly indices 2007 – 2009; growth rates)

quarter to quarter
growth rates



correlations
(**growth** rates)

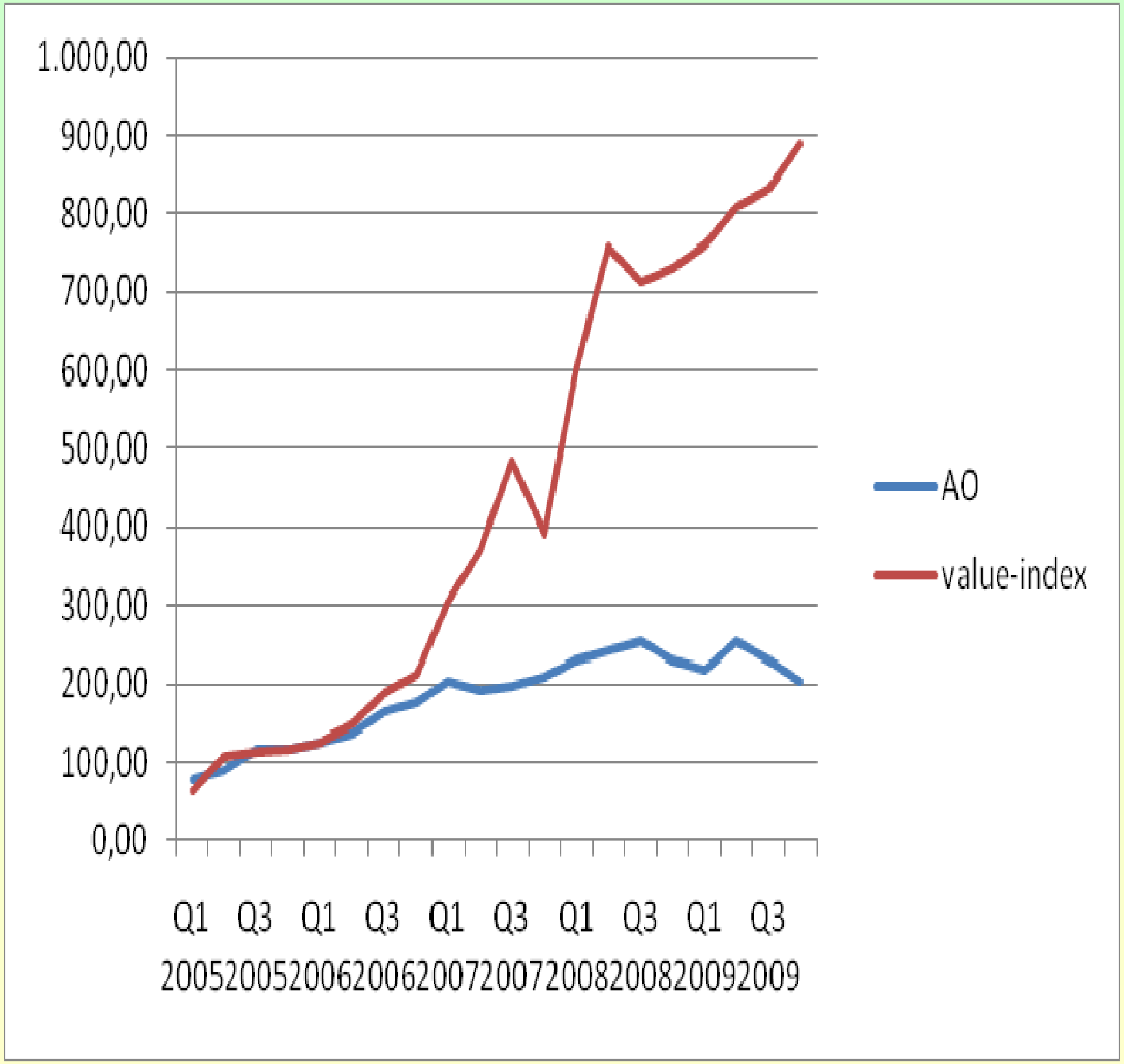
	AO	***QO***	***OY***	***CP***
A0	1,0000			
QO	0,9938	1,0000		
OY	0,9347	0,9181	1,0000	
CP	0,9330	0,9352	0,9128	1,00

in the **levels**

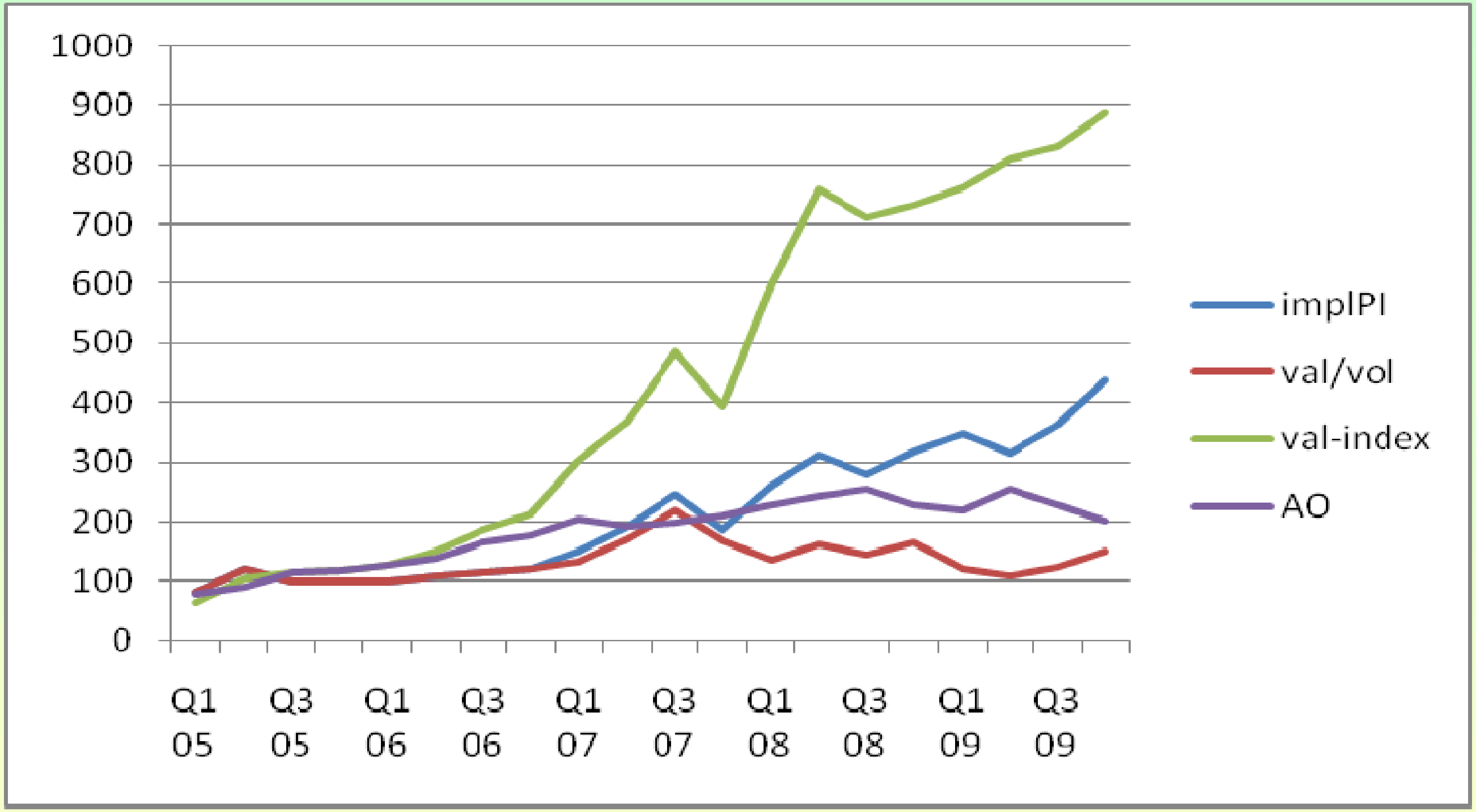
	AO	**QO**	**OY**
QO	0,99325	1	
OY	0,95946	0,95795	1
CD	0,97731	0,97061	0,95930

7.4.1 (5) Time series of values, the value index and the quarterly volume indices

q, y	value	value-index
Q1 2005	**300**	**63,32**
Q2 2005	**505**	**106,60**
Q3 2005	**540**	**113,98**
Q4 2005	**550**	**116,09**
Q1 2006	**600**	**126,65**
Q2 2006	**710**	**149,87**
Q3 2006	**890**	**187,86**
Q4 2006	**1010**	**213,19**
Q1 2007	**1440**	**303,96**
Q2 2007	**1755**	**370,45**
Q3 2007	**2300**	**485,49**
Q4 2007	**1860**	**392,61**
Q1 2008	**2840**	**599,47**
Q2 2008	**3590**	**757,78**
Q3 2008	**3370**	**711,36**
Q4 2008	**3460**	**730,34**
Q1 2009	**3610**	**762,01**
Q2 2009	**3830**	**808,44**
Q3 2009	**3940**	**831,66**
Q4 2009	**4210**	**888,65**



7.4.1 (6) Time series of the value index and the quarterly volume indices



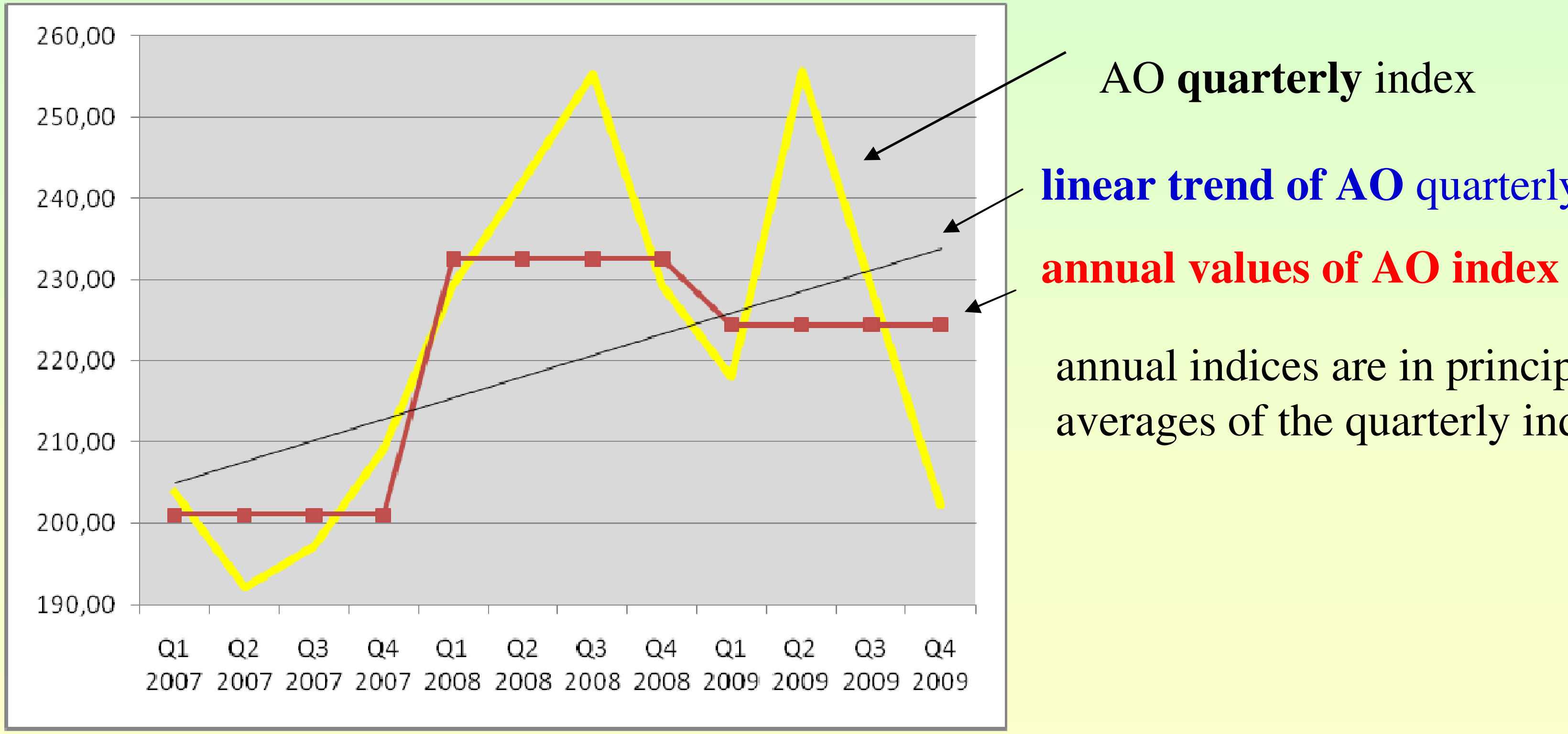
val-index = value index (from 63.32 to 888.65)

val/vol = value divided by volumes at average prices of the previous year

implPI = implicit price indes (= value index divided by AO volume index)

7.4.2 (1) **Annual** indices: Synopsis of methods and volumes at constant prices

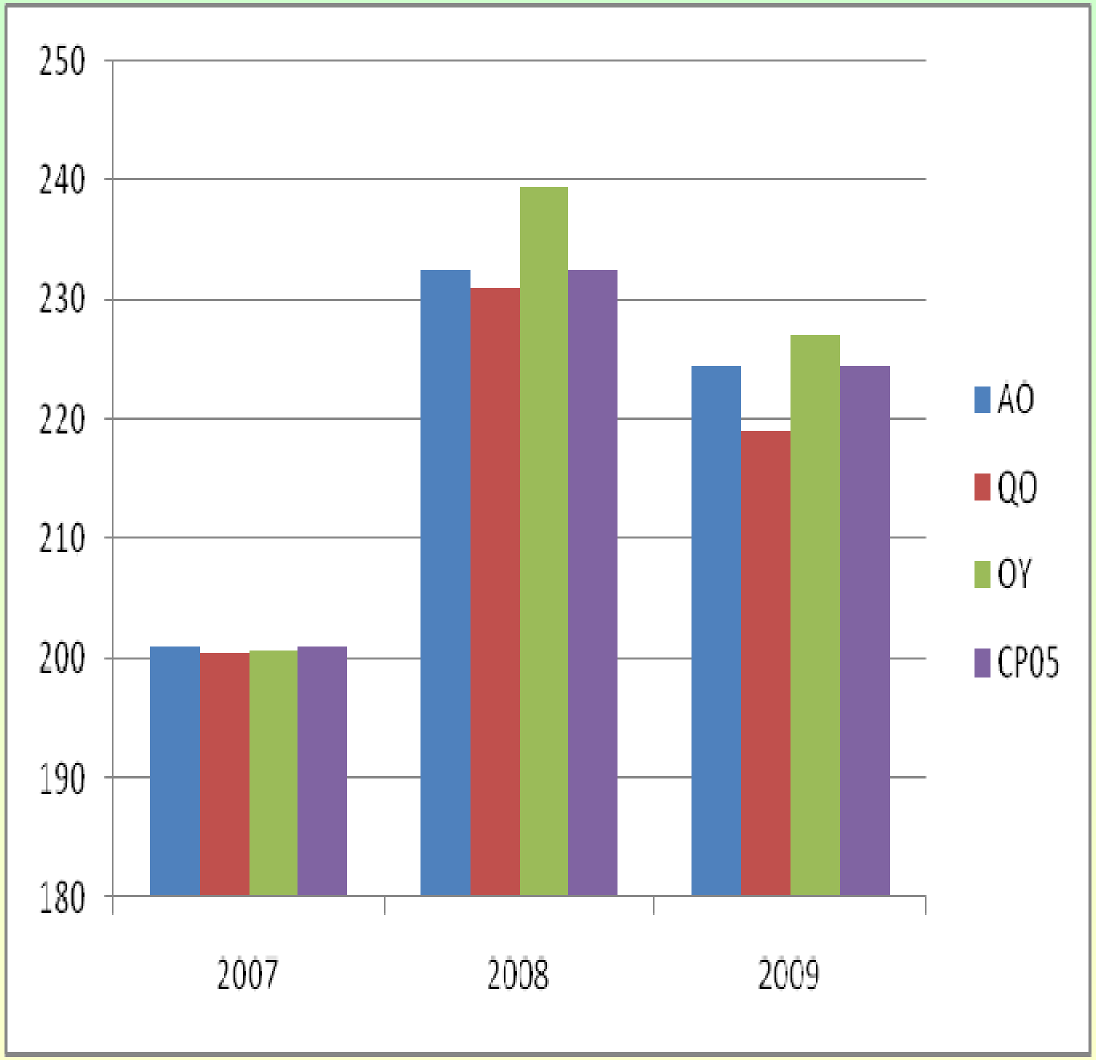
y	volumes*	index*	volumes**	index**	AO	QO	OY
06	716.81	151.30	716.81	151.30	151.30	151.30	151.30
07	952.56	201.07	1064.05	224.60	201.07	200.49	200.65
08	1101.86	232.58	2190.30	462.35	232.58	231.06	239.33
09	1063.31	224.44	3138.22	662.42	224.44	218.96	227.06



annual indices are in principle only averages of the quarterly indices

* at constant average prices of 2005
** at average prices of the preceding year

7.4.2 (2) Graph of annual indices



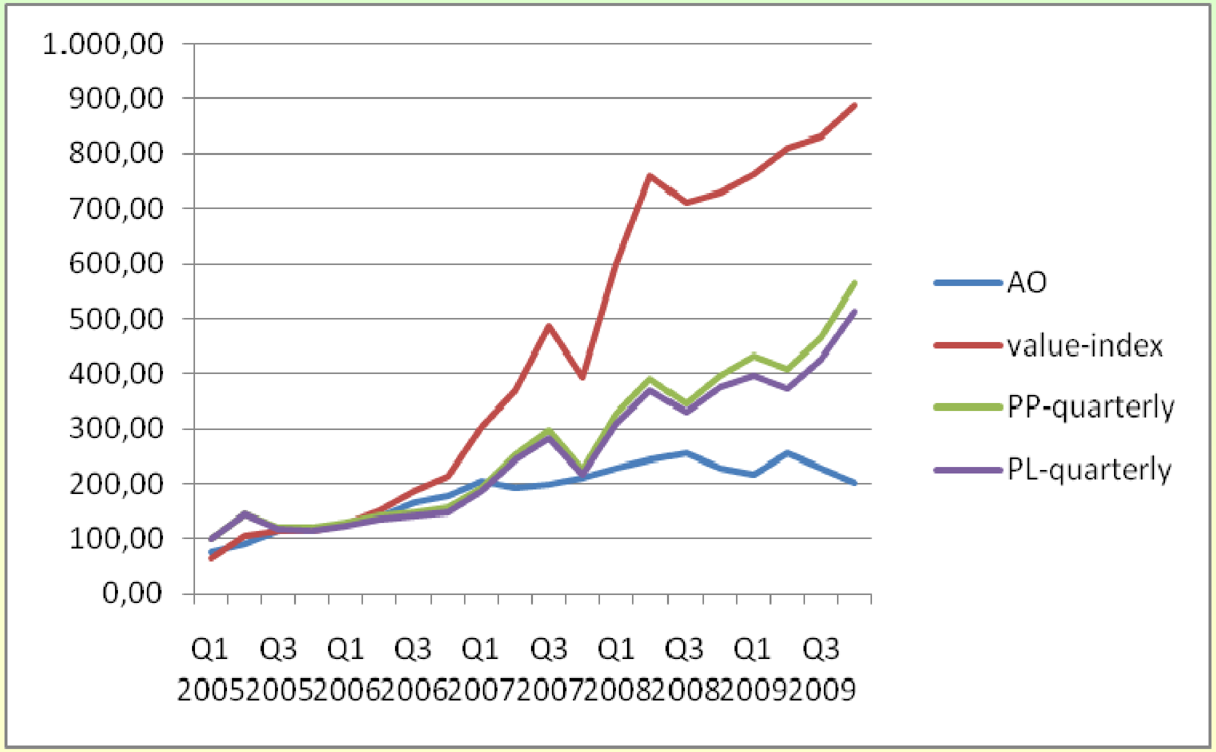
The differences between the three methods and constant prices at 2005 are much smaller than expected

7.4.3 (1) Quarterly chained quantity indices *(or volume indices)*

$$\overline{Q}^{LQC}_{00,11} = \frac{\sum_i p_{i,0,0} q_{i,0,1}}{\sum_i p_{i,0,0} q_{i,0,0}} \cdot \frac{\sum_i p_{i,0,1} q_{i,0,2}}{\sum_i p_{i,0,1} q_{i,0,1}} \cdot \frac{\sum_i p_{i,0,2} q_{i,0,3}}{\sum_i p_{i,0,2} q_{i,0,2}} \cdot \frac{\sum_i p_{i,0,3} q_{i,0,4}}{\sum_i p_{i,0,3} q_{i,0,3}} \cdot \frac{\sum_i p_{i,0,4} q_{i,1,1}}{\sum_i p_{i,0,4} q_{i,0,4}}$$

$$\overline{Q}^{LQC}_{00,21} = \overline{Q}^{LQC}_{00,11} \frac{\sum p_{i11} q_{i12}}{\sum p_{i11} q_{i11}} \frac{\sum p_{i12} q_{i13}}{\sum p_{i12} q_{i12}} \frac{\sum p_{i13} q_{i14}}{\sum p_{i13} q_{i13}} \frac{\sum p_{i14} q_{i21}}{\sum p_{i14} q_{i14}}$$

chained **price** indices defined analogously

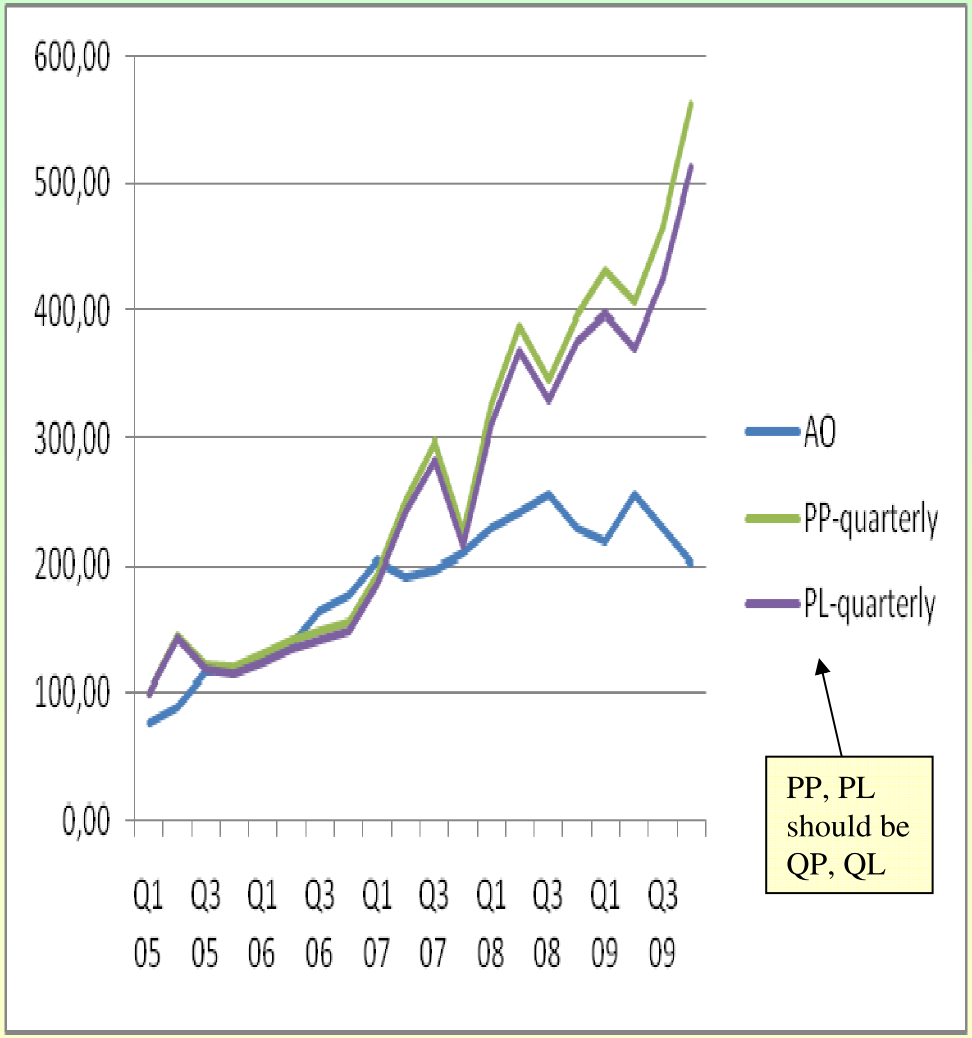


The quarterly chained quantity indices of Laspeyres (see formula above) Q^{LQC} or Paasche index Q^{PQC} are in between the value index and the AO index (and therefore also the QO and OY index)

here and in the following slides I made a mistake with the symbols: PP and PL should read QP and QL

7.4.3 (2) Quarterly chained quantity (volume) indices Q^{LQC}, Q^{PQC}

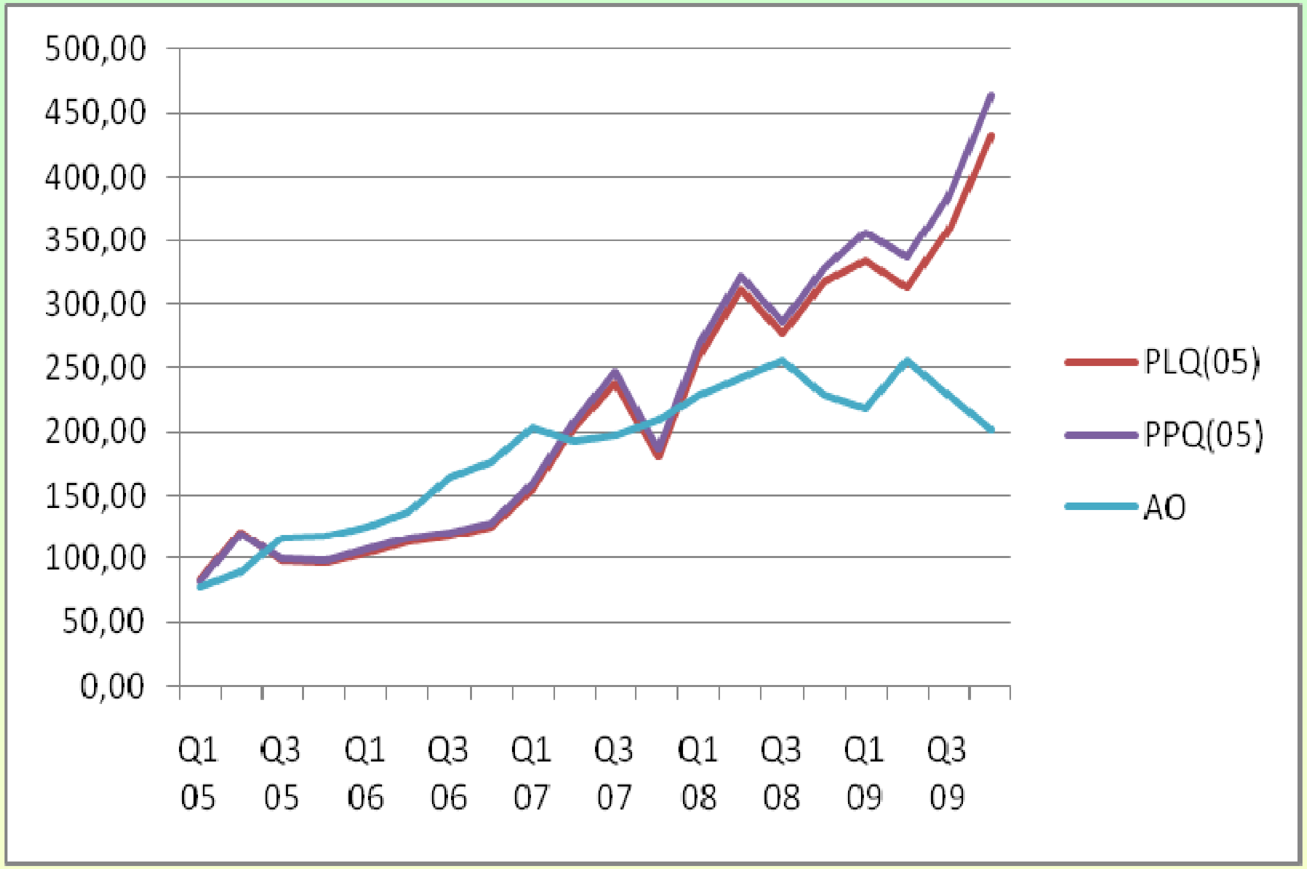
q,y	AO	Paasche quarterly	Laspeyres quarterly
Q1 05	**77,16**	**100,00**	**100,00**
Q2 05	**89,35**	**144,29**	**143,33**
Q3 05	**115,74**	**120,80**	**117,79**
Q4 05	**117,75**	**119,71**	**114,52**
Q1 06	**125,92**	**130,59**	**123,89**
Q2 06	**138,11**	**141,56**	**134,21**
Q3 06	**164,50**	**147,35**	**140,83**
Q4 06	**176,69**	**155,84**	**148,74**
Q1 07	**203,98**	**193,45**	**185,55**
Q2 07	**192,07**	**251,49**	**242,25**
Q3 07	**197,25**	**298,16**	**282,97**
Q4 07	**209,16**	**226,36**	**215,30**
Q1 08	**229,22**	**326,32**	**309,64**
Q2 08	**242,24**	**389,85**	**369,61**
Q3 08	**255,26**	**346,65**	**328,94**
Q4 08	**229,22**	**397,15**	**376,77**
Q1 09	**218,00**	**431,84**	**397,46**
Q2 09	**255,65**	**407,38**	**371,04**
Q3 09	**228,99**	**466,59**	**425,29**
Q4 09	**202,32**	**562,85**	**513,80**



$Q^{PQC} > Q^{LQC}$ in this numerical example

7.4.3 (3) Quarterly chained indices rebased Ø2005 = 100 instead of Q1 05 = 100

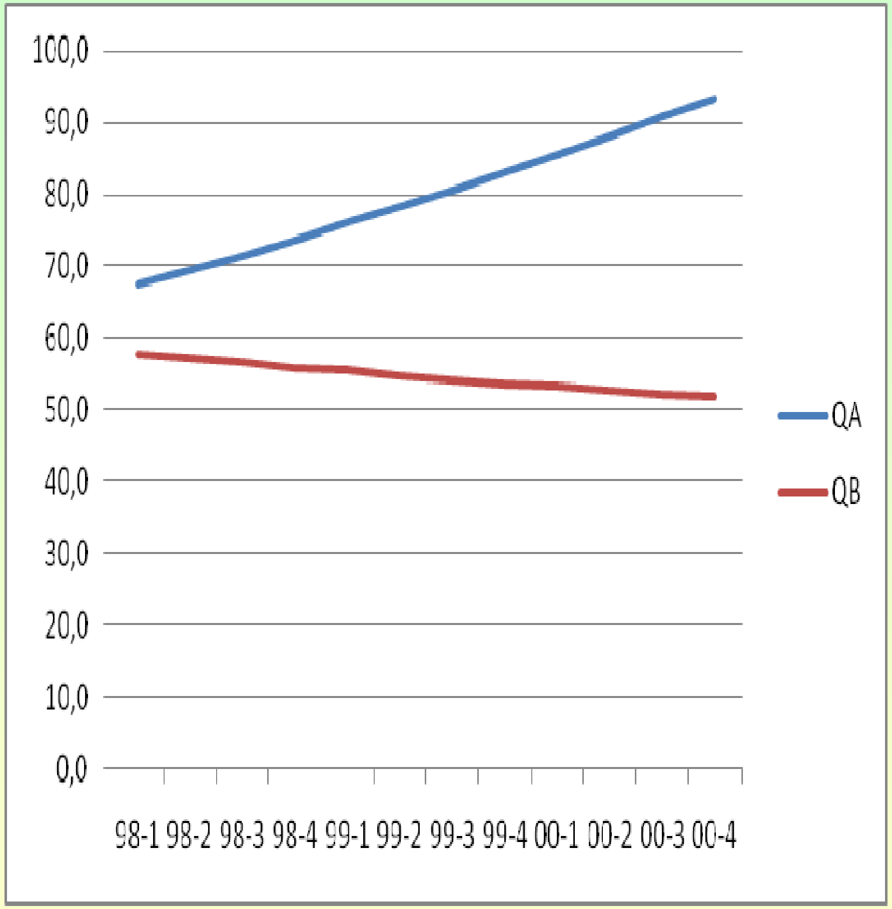
	PLQ(05)	PPQ(05)
Q1 05	**84,10**	**82,51**
Q2 05	**120,54**	**119,05**
Q3 05	**99,06**	**99,67**
Q4 05	**96,31**	**98,77**
Q1 06	**104,19**	**107,75**
Q2 06	**112,87**	**116,80**
Q3 06	**118,43**	**121,58**
Q4 06	**125,08**	**128,58**
Q1 07	**156,05**	**159,62**
Q2 07	**203,73**	**207,50**
Q3 07	**237,97**	**246,01**
Q4 07	**181,07**	**186,77**
Q1 08	**260,40**	**269,25**
Q2 08	**310,83**	**321,66**
Q3 08	**276,63**	**286,02**
Q4 08	**316,86**	**327,69**
Q1 09	**334,26**	**356,31**
Q2 09	**312,03**	**336,13**
Q3 09	**357,66**	**384,98**
Q4 09	**432,09**	**464,41**



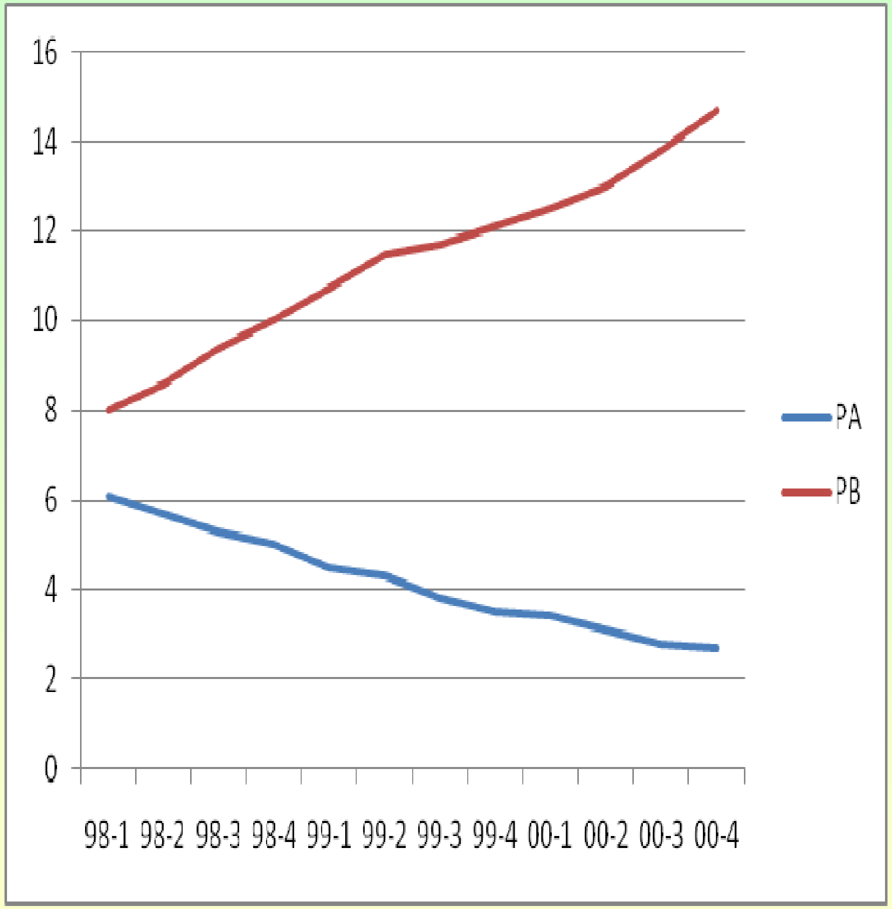
average of year 2005 = 100 instead of first quarter of 2005 = 100
Again quarterly chained indices ($Q^{PQC} > Q^{LQC}$) are rising much higher than annually chained indices (of AO type)

7.4.4 (1) Numerical example in the IMF manual (1)

quantities

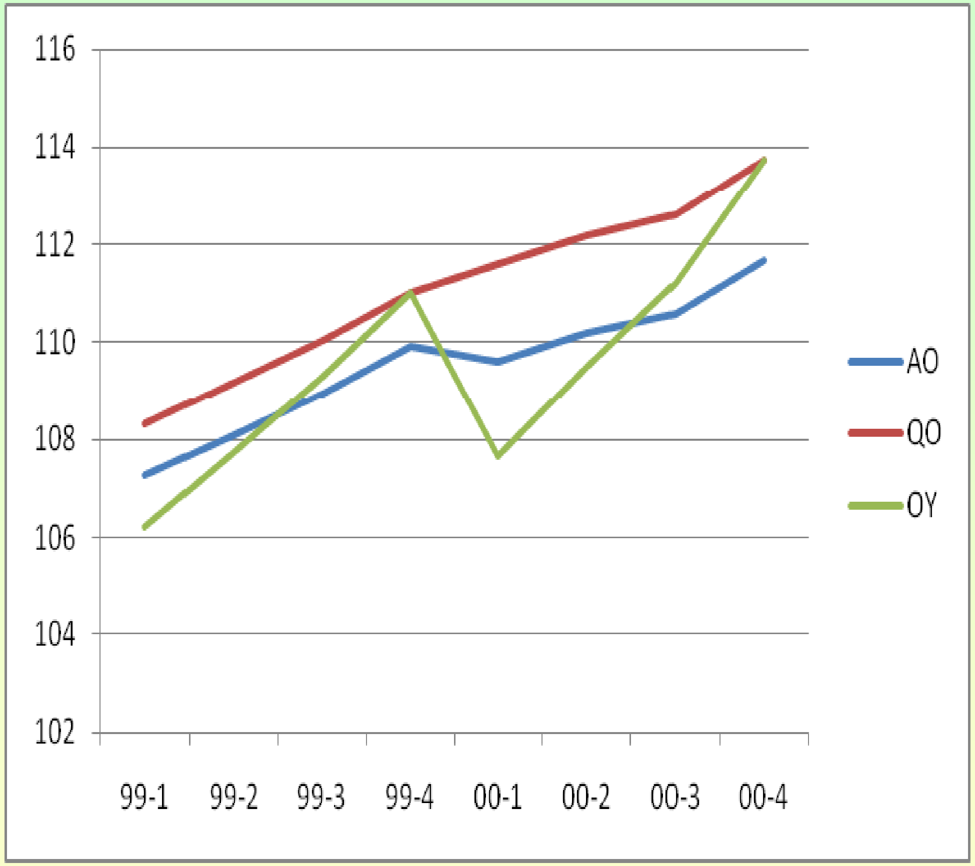


prices

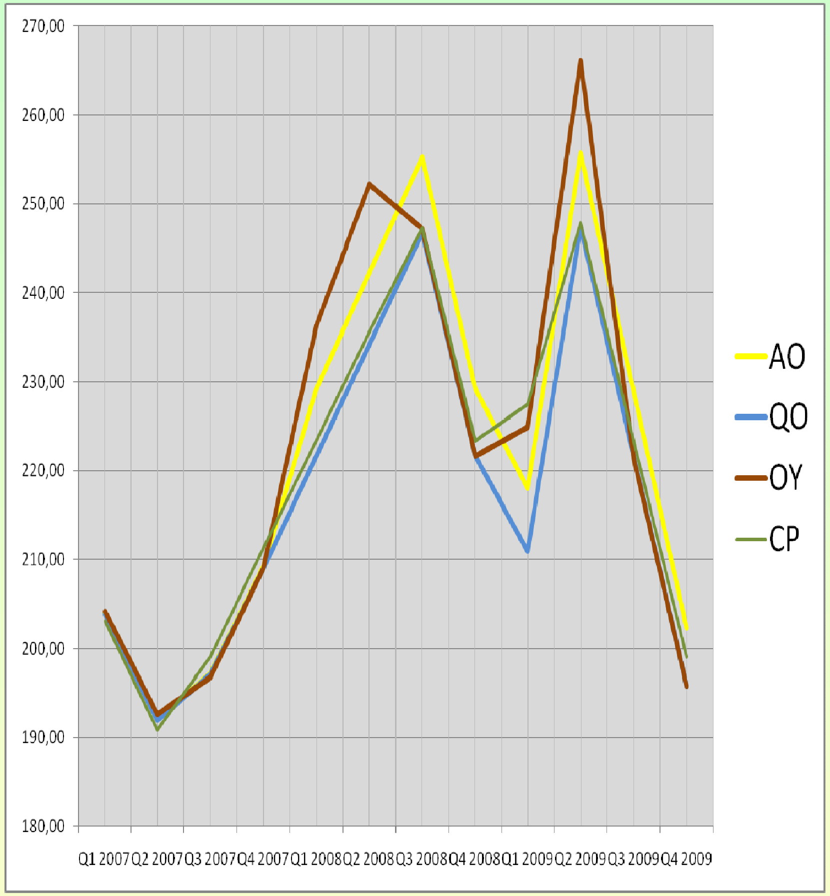


7.4.4 (2) Numerical example in the IMF manual (2)

quarterly indices
(only two years 1999 and 2000 are different



our example slide 38



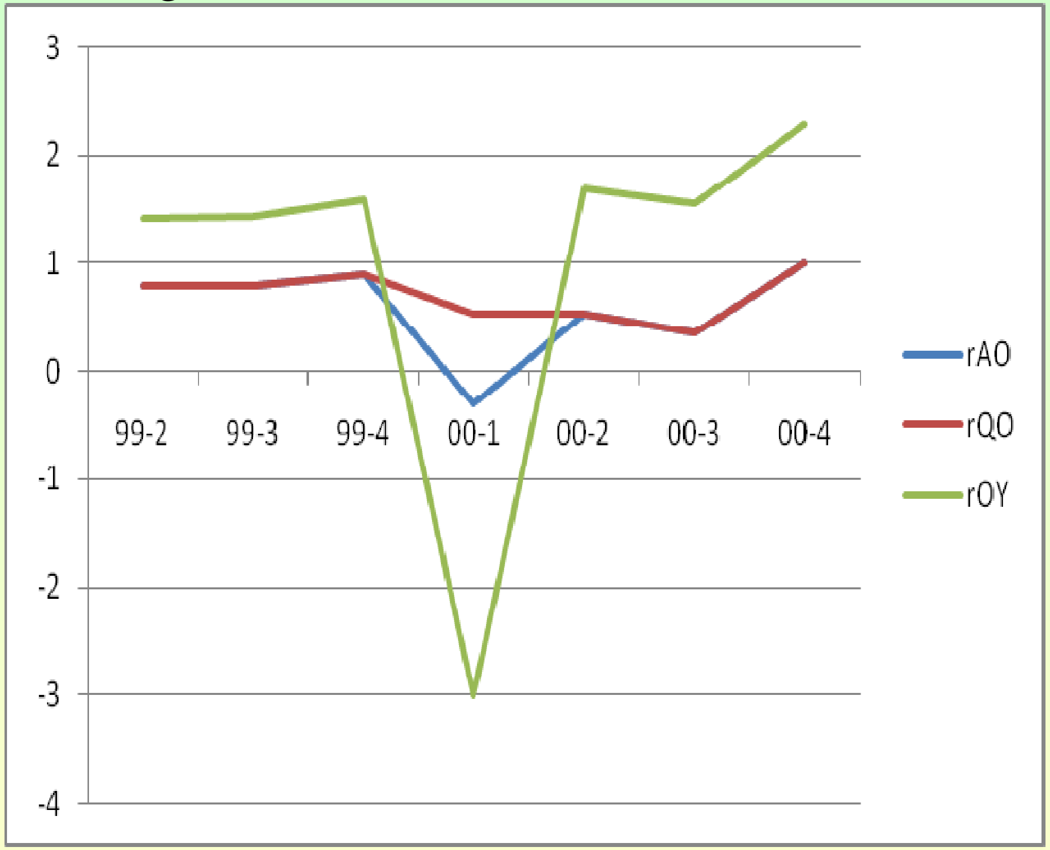
correlations

	AO	QO
QO	0,98345	1,00000
OY	0,90783	0,82966

	AO	QO
QO	0,99325	1
OY	0,95946	0,95795

7.4.4 (3) Numerical example in the IMF manual (3)

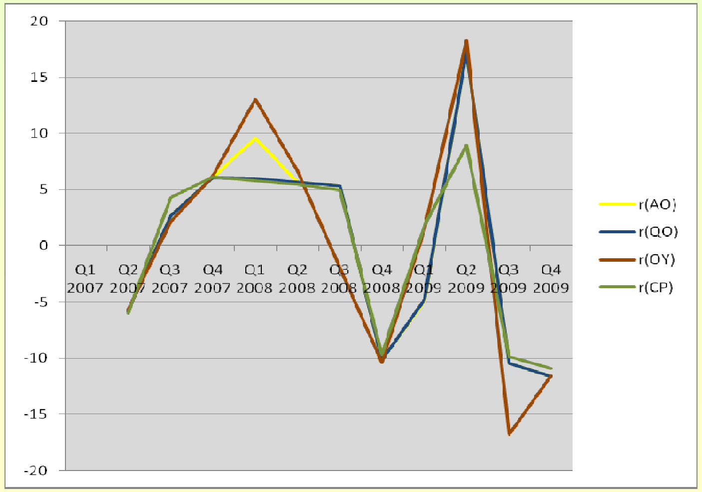
The three methods AO, QO and OY: growth rates



	rAO	rQO	rOY
99-2	0,7831	0,7940	1,4120
99-3	0,7863	0,7878	1,4388
99-4	0,8995	0,8907	1,5831
00-1	-0,3002*	0,5315*	-3,0087
00-2	0,5292	0,5287	1,6904
00-3	0,3630	0,3655	1,5618
00-4	1,0038	1,0036	2,2752

* difference is indicating a "drift"

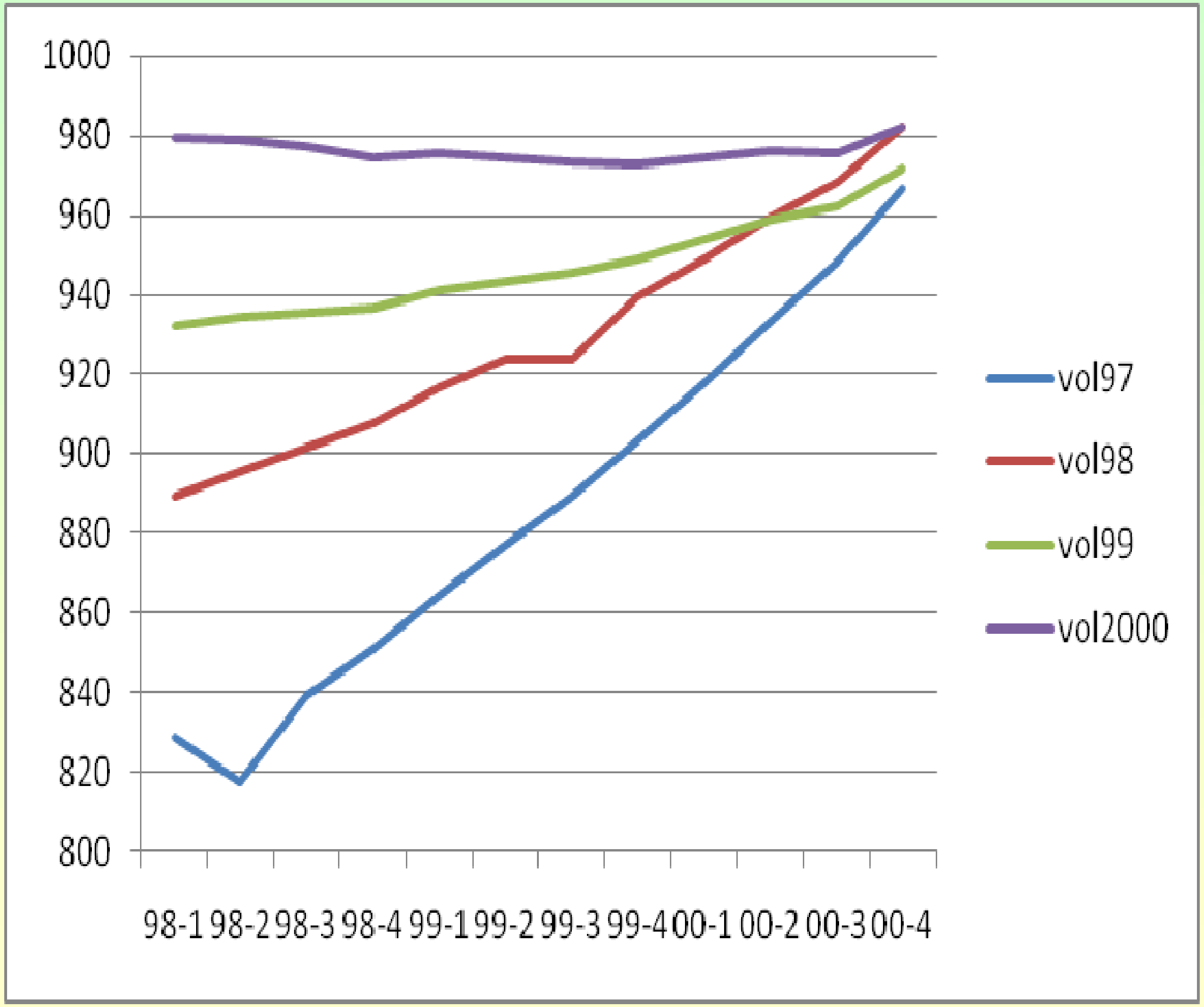
our example (slide 38) ⇒
again OY an exception



7.4.4 (4) Numerical example in the IMF manual (4)

volumes (absolute figures)
at constant prices of 97, 98, and 2000

the issue of re-writing of history with CP deflation



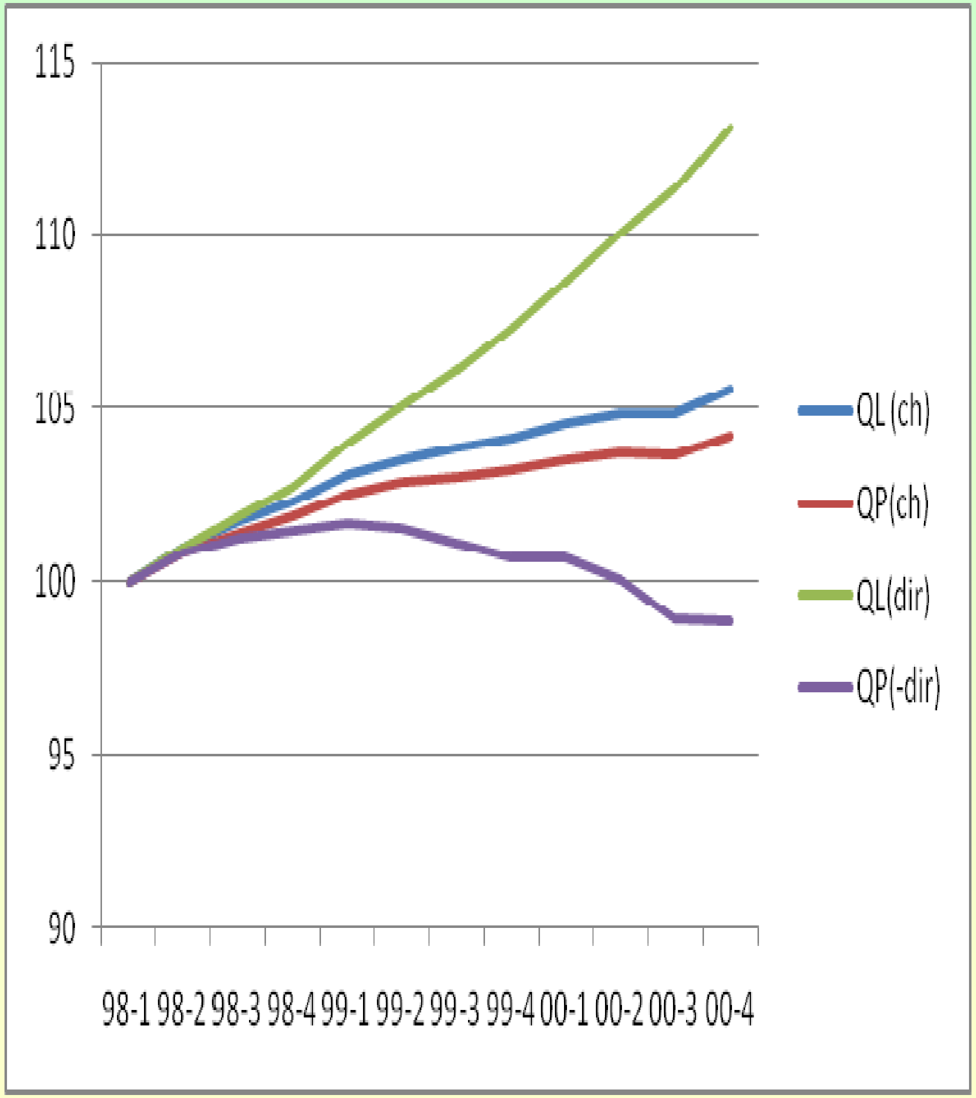
underlying prices
(unit values)

	p_a	p_b
97	7.0	6.0
98	5.5	9.0
99	4.0	11.5
00	3,0	13,5

The problem with the CP-approach:

figures depend on which year is taken as basis for the constant prices volumes

7.4.4 (5) IMF manual example: Quarterly chained and direct quantity (volume) indices



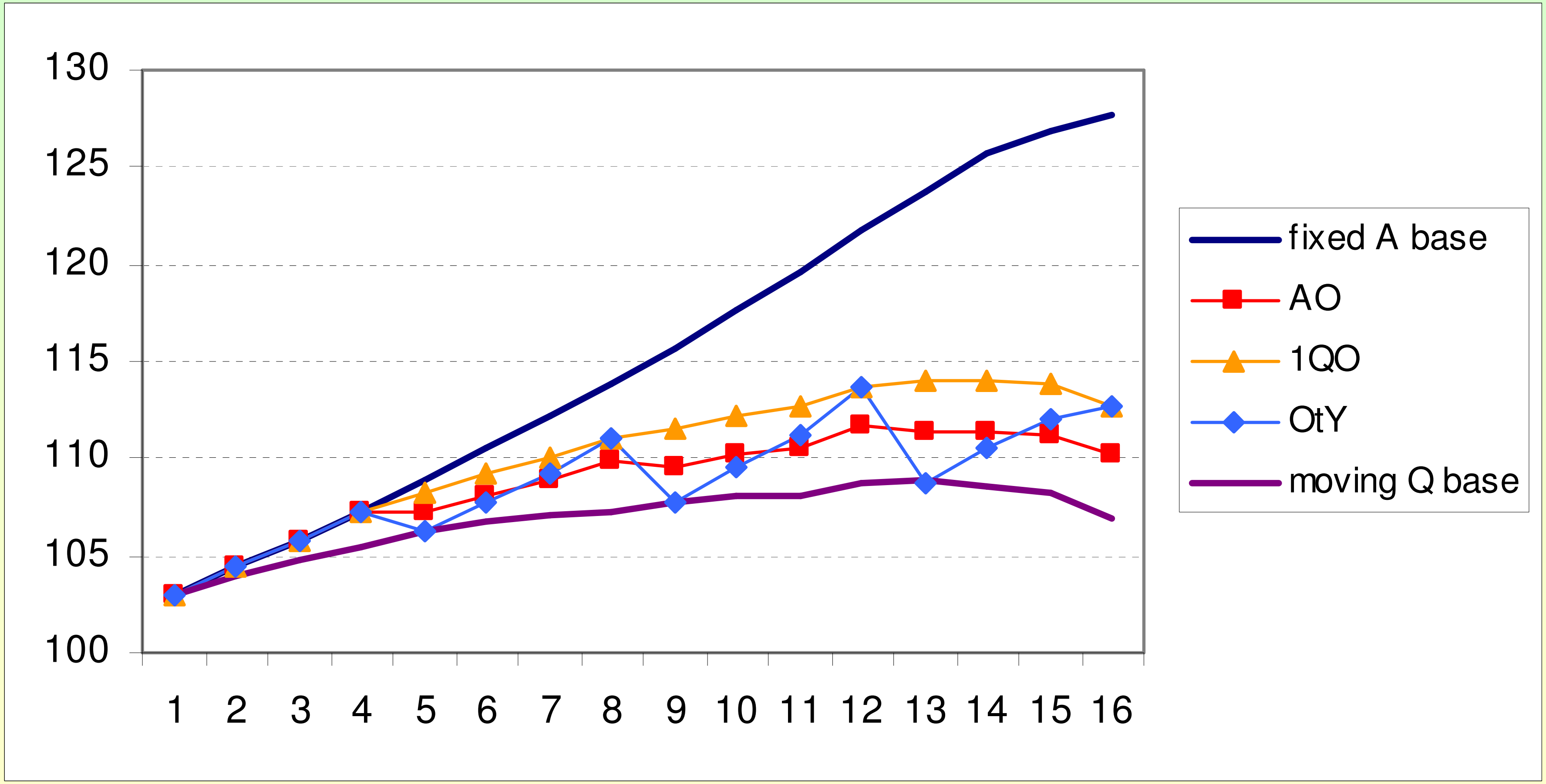
	QL(ch)	QP(ch)	QL(dir)	QP(-dir)
98-1	100	100	100	100
98-2	100,94	100,81	100,94	100,81
98-3	101,72	101,42	101,86	101,27
98-4	102,28	101,86	102,76	101,48
99-1	103,11	102,52	104,00	101,65
99-2	103,54	102,84	105,06	101,54
99-3	103,87	103,03	106,12	101,12
99-4	104,14	103,19	107,31	100,70
00-1	104,53	103,54	108,63	100,69
00-2	104,85	103,74	110,06	100,08
00-3	104,88	103,63	111,32	98,94
00-4	105,55	104,23	113,12	98,85

The result comes up to our expectations

1. Strong substitution effect

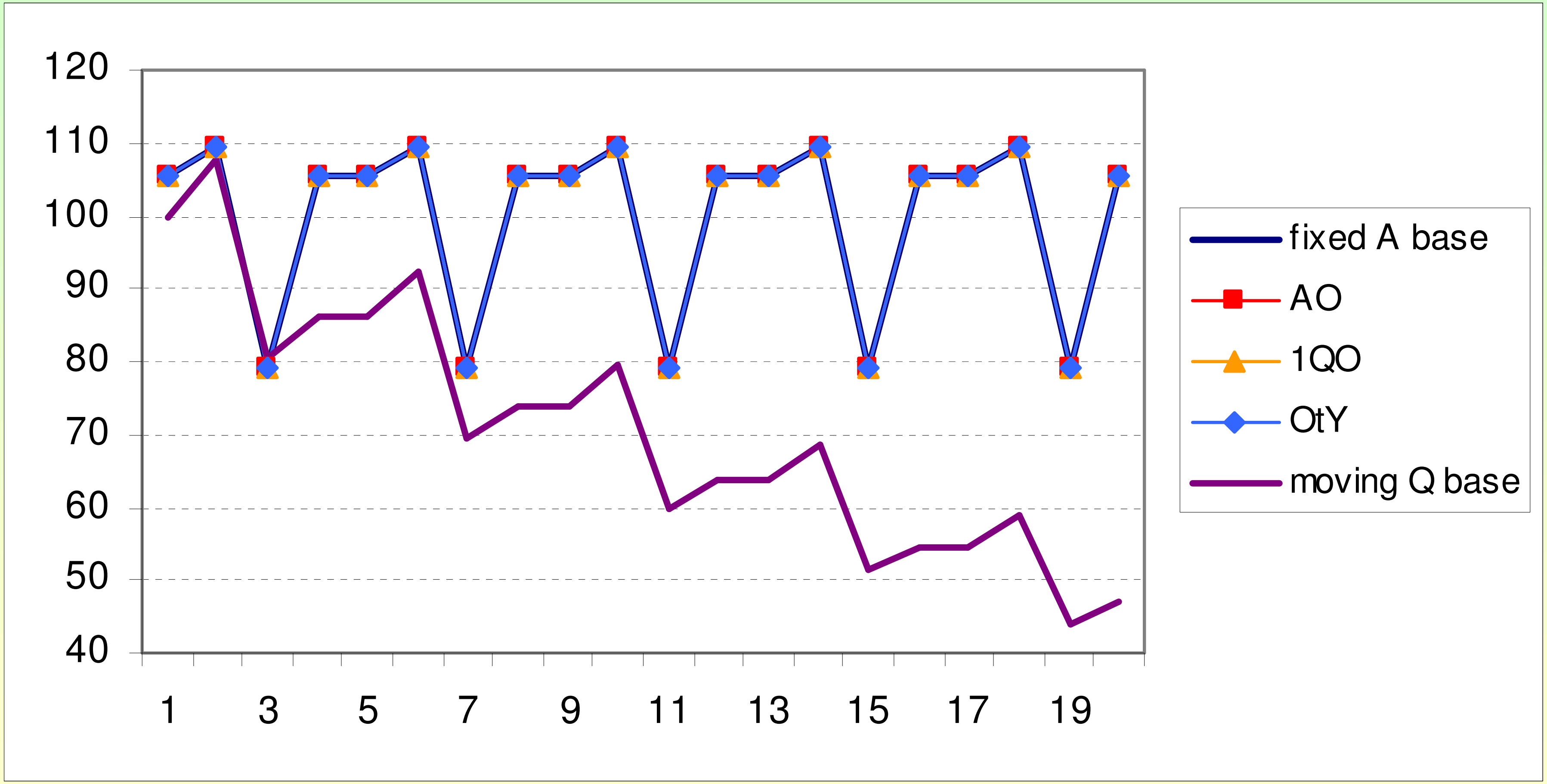
* reproduced here with the permission of Dr. Ingo Kuhnert

Quarterly volume index series (all Laspeyres): a fixed-base index (with annual base), a moving base one with quarterly re-weighting, and three annually re-weighted chain-linked series using different linking techniques. Source data contains a **strong substitution effect**.



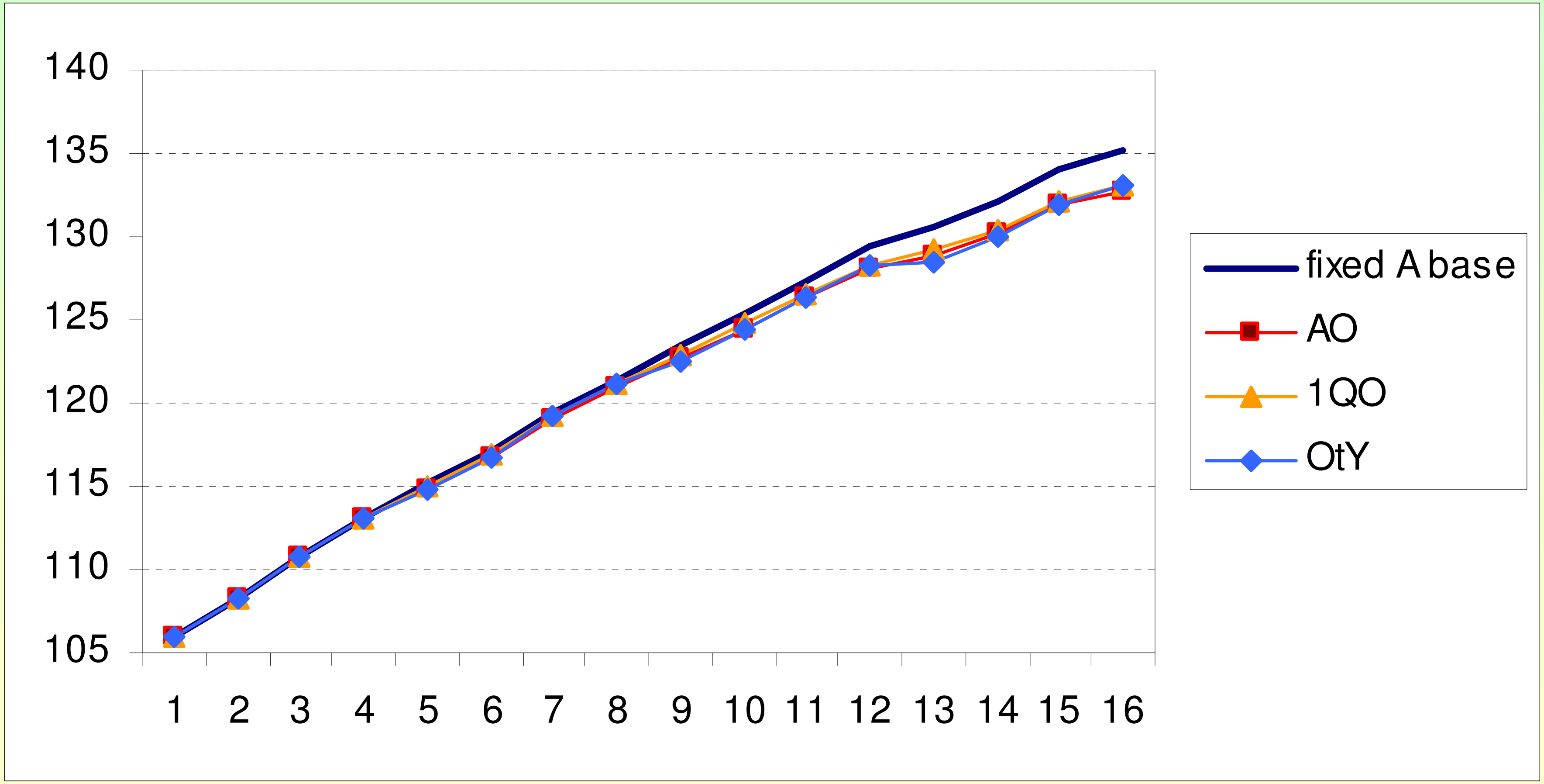
2. Cycle and no trend

Quarterly volume index series (all Laspeyres): a fixed-base index (with annual base), a moving base one with quarterly re-weighting, and three annually re-weighted chain-linked series using different linking techniques. Source data contains a **constant seasonal cycle and no trend**.



3. Trend, weak substitution effect

Quarterly volume index series (all Laspeyres): a fixed-base index (with annual base), a moving base one with quarterly re-weighting, and three annually re-weighted chain-linked series using different linking techniques. Source data contains a **trend and weak substitution effect**.



This section contains another look at the formulas to

1. see which comparisons can consistently be made (interpretation of a sequence of indices, consistency between QNA and ANA)
2. if percentage changes of the indices can reasonably be decomposed into growth rates of "components" and if these growth rates are comparable over time

Its purpose is to prepare a final assessment of the three techniques (see sec. **7.6**)

7.5.1 (1) Time series and comparisons: AO method

(19) sequence of annual indices

$$I_y^{AO} = \frac{\sum\sum \bar{p}_{i0} q_{i1q}}{W_0} \cdot \frac{\sum\sum \bar{p}_{i1} q_{i2q}}{W_1} \cdot \ldots \cdot \frac{\sum\sum \bar{p}_{i,y-1} q_{i,y,q}}{W_{y-1}}$$

(20) sequence of quarterly indices

$$I_{y,q=1}^{AO} = I_{y-1}^{AO} \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=1}}{W_{y-1}/4} \qquad I_{y,q=2}^{AO} = I_{y-1}^{AO} \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=2}}{W_{y-1}/4}$$

(21) comparison D1 (y,q) → (y,q-1)

$$\frac{I_{y,q}^{AO}}{I_{y,q-1}^{AO}} = \frac{\bar{V}_{y,y-1,q}}{\bar{V}_{y,y-1,q-1}} = \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q}}{\sum_i \bar{p}_{i,y-1} q_{i,y,q-1}}$$

pure comparison!!

(22) comparison D2 (y,q) → (y-1,q)

$$\frac{I_{y,q}^{AO}}{I_{y-1,q}^{AO}} = \frac{\bar{V}_{y.y-1,q}}{\bar{V}_{y-1,y-2,q}} \cdot \frac{\sum_q \bar{V}_{y-1,y-2,q}}{W_{y-1}} = Q_{y-1,q}^{y,q} \cdot A_{y-1,y-1}^{y-1,y-2}$$

→

(23) comparison D3 (y,q=4) → (y+1,q=1)

$$\frac{I_{y+1,q=1}^{AO}}{I_{y,q=4}^{AO}} = \frac{\bar{V}_{y+1,y,q=1}}{\bar{V}_{y,y-1,q=4}} \cdot \frac{\sum_q \bar{V}_{y,y-1,q}}{W_y} = Q_{y,4}^{y+1,1} A_{y,y}^{y,y-1}$$

→

Eq. 21 shows: **pure comparison of successive quarters of the same year**; they only differ from one another with respect to quantities

7.5.1 (2) Alternative presentation of eqs. (19), (20) AO method

y	q = 1	…	q = 4	annual **AO** index (y)
0	$I_{0,1}^{AO} = \frac{\bar{V}_{0,0,q=1}}{\frac{1}{4}W_0}$		$I_{0,4}^{AO} = \frac{\bar{V}_{0,0,q=4}}{\frac{1}{4}W_0}$	$\frac{\sum_q \bar{V}_{0,0,q}}{W_0} = \frac{1}{4}\sum_q I_{0,q}^{AO} = I_0^{AO} = 1$
1	$I_{1,1}^{AO} = \frac{\bar{V}_{1,0,q=1}}{\frac{1}{4}W_0}$		$I_{1,4}^{AO} = \frac{\bar{V}_{1,0,q=4}}{\frac{1}{4}W_0}$	$I_1^{AO} = \frac{\sum_q \bar{V}_{1,0,q}}{W_0}$
2	$I_{2,1}^{AO} = I_1^{AO} \frac{\bar{V}_{2,1,q=1}}{\frac{1}{4}W_1}$		$I_{2,1}^{AO} = I_1^{AO} \frac{\bar{V}_{2,1,q=4}}{\frac{1}{4}W_1}$	$I_2^{AO} = \frac{\sum_q \bar{V}_{1,0,q}}{W_0} \frac{\sum_q \bar{V}_{2,1,q}}{W_1}$
3	$I_{3,1}^{AO} = I_2^{AO} \frac{\bar{V}_{3,2,q=1}}{\frac{1}{4}W_2}$		$I_{3,4}^{AO} = I_2^{AO} \frac{\bar{V}_{3,2,q=4}}{\frac{1}{4}W_2}$	$I_3^{AO} = \frac{\sum_q \bar{V}_{1,0,q}}{W_0} \frac{\sum_q \bar{V}_{2,1,q}}{W_1} \frac{\sum_q \bar{V}_{3,2,q}}{W_2}$

annual indices are forming a chain (product of links)

$$I_y^{AO} = \frac{\sum_q \bar{V}_{1,0,q}}{W_0} \frac{\sum_q \bar{V}_{2,1,q}}{W_1} \frac{\sum_q \bar{V}_{3,2,q}}{W_2} \cdots \frac{\sum_q \bar{V}_{y,y-1,q}}{W_{y-1}}$$

7.5.1 (3) Interpretations of eqs. 22 and 23 (AO comparisons between different years)

22* $$\frac{I^{AO}_{y,q}}{I^{AO}_{y-1,q}} = \frac{\overline{V}_{y.y-1,q}}{\overline{V}_{y-1,y-2,q}} \Big/ \frac{W_{y-1}}{\sum_q \overline{V}_{y-1,y-2,q}} = Q^{y,q}_{y-1,q} \div A^{y-1,y-1}_{y-1,y-2}$$

A = annual index
Q = quarterly index

Q is a quarter specific ratio (reflecting volume change, however at different prices).

Numerator and denominator differ with respect to **both**, (average) prices *and* quantities.

Hence **in 22 the comparison is biased** (the same is true for 23)

A^{-1} is a **Paasche price index** relating prices in y-1 to those in y-2, and A may be viewed as (partially) **correcting the bias**.

In A numerator and denominator differ with respect to prices only.

23* $$\frac{I^{AO}_{y+1,q=1}}{I^{AO}_{y,q=4}} = \frac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y-1,q=4}} \cdot \frac{\sum_q \overline{V}_{y,y-1,q}}{W_y} = Q^{y+1,1}_{y,4} \div A^{y,y}_{y,y-1}$$

again Q does **not** provide a **pure** comparison of volumes

$$A^{y,y}_{y,y-1} = \frac{\sum_q \overline{V}_{y,y,q}}{\sum_q \overline{V}_{y,y-1,q}}$$

and dividing by the Paasche price index A (comparing average prices of y and y-1 on the basis of quantities of y) amounts to making a correction for the different prices in Q

* counterparts in the QO case **(22) → (27)**, and **(23) → (28)**

However, a pure quantity comparison between y+1, q = 1 and y, q = 4 would be

23a $$D = \frac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y,q=4}} \neq Q_{y,4}^{y+1} = \frac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y-1,q=4}}$$ for $Q_{y,4}^{y+1}$ see eq. 23

or $$D = \sum_i \overline{p}_{iy} q_{i,y+1,q=1} \Big/ \sum_i \overline{p}_{iy} q_{i,y,q=4}$$

the "contamination"* of the comparison now may be viewed as a relation between two Paasche price indices

23b $$\text{cont} = \frac{I_{y+1,q=1}^{AO} \Big/ I_{y,q=4}^{AO}}{D} = \frac{\overline{V}_{y,y,q=4} \Big/ \overline{V}_{y,y-1,q=4}}{\sum_q \overline{V}_{y,y} \Big/ \sum_q \overline{V}_{y,y-1}}$$

(eq. 23 → $I_{y,q=4}^{AO}$; correct → D)

no bias (cont = 1) if the price movement y-1→ y in q=4 equals the (average) annual price change

$$\text{cont} = \frac{I_{y+1,q=1}^{AO} \Big/ I_{y,q=4}^{AO}}{D} = \frac{\sum_i \overline{p}_{iy} q_{i,y,q=4} \Big/ \sum_i \overline{p}_{i,y-1} q_{i,y,q=4}}{\sum_q \sum_i \overline{p}_{iy} q_{i,y,q} \Big/ \sum_q \sum_i \overline{p}_{i,y-1} q_{i,y,q}}$$

in other words: if q = 4 is representative of the whole year

* Robert Kirchner, Deutsche Bundesbank, June 2006

7.5.1x (1) Digression: Contribution of aggregates to percentage change of the volume

AO Method y,q → y,q+1 "no problem" (Tödter*) because of the same average prices (however, the weights are changing, due to different quantities in the successive quarters)

General formula

$$g_{y,q+1}^{AO} = \frac{I_{y,q+1}^{AO} - I_{y,q}^{AO}}{I_{y,q}^{AO}} = \frac{\overline{V}_{y,y-1,q+1} - \overline{V}_{y,y-1,q}}{\overline{V}_{y,y-1,q}} = \frac{\sum_i \overline{p}_{i,y} q_{i,y,q+1} - \sum_i \overline{p}_{i,y} q_{i,y,q}}{\sum_i \overline{p}_{i,y} q_{i,y,q}}$$

$$g_{y,q+1}^{AO} = \frac{\overline{p}_{A,y-1} q_{A,y,q}}{\sum \overline{p}_{i,y-1} q_{i,y,q}} \left(\frac{q_{A,y,q+1} - q_{A,y,q}}{q_{A,y,q}} \right) + \frac{\overline{p}_{B,y-1} q_{B,y,q}}{\sum \overline{p}_{i,y-1} q_{i,y,q}} \left(\frac{q_{B,y,q+1} - q_{B,y,q}}{q_{B,y,q}} \right)$$

$$= w_{Ayq} \left(\frac{q_{A,y,q+1} - q_{A,y,q}}{q_{A,y,q}} \right) + w_{Byq} \left(\frac{q_{B,y,q+1} - q_{B,y,q}}{q_{B,y,q}} \right)$$

weights are **not** constant; aggregation is not "no problem"

weights variable (depending on y and q)

$$w_{Ayq} = \frac{\overline{p}_{A,y-1} q_{A,y,q}}{\sum \overline{p}_{y-1} q_{y,q}} \qquad w_{Byq} = 1 - w_{Ayq}$$

*) "Die Zerlegung des Gesamtwachstums in die Wachstumsbeiträge der Komponenten innerhalb eines Jahres ist **unproblematisch**" (p. 18)

7.5.1x (2) Digression: Example: 2007,2 → 2007,3 and 2007,3 → 2007,4

Average **prices** in 2006	of A **12.63**	of B **81**
quarter	**quantities**	
2	**55**	**4**
3	**70**	**2**
4	**75**	**2**
	weights	
quarter	**wA**	**wB**
2	**0.6820**	**0.3180**
3	**0.8452**	**0.1247**

2007, 2 → 2007, 3

$$g = \frac{I^{AO}_{y,q+1} - I^{AO}_{y,q}}{I^{AO}_{y,q}} = \frac{\overline{V}_{y,y-1,q+1} - \overline{V}_{y,y-1,q}}{\overline{V}_{y,y-1,q}}$$

$$g = \frac{197.25 - 192.07}{192.07} = \frac{1046.21 - 1018.74}{1018.74} = 0.02697$$

$$= w_{Ayq}\left(\frac{q_{A,y,q+1} - q_{A,y,q}}{q_{A,y,q}}\right) + w_{Byq}\left(\frac{q_{B,y,q+1} - q_{B,y,q}}{q_{B,y,q}}\right)$$

$$= \frac{12.63 \cdot 55}{1018.74}\left(\frac{70-55}{55}\right) + \frac{81 \cdot 4}{1018.74}\left(\frac{2-4}{4}\right) = 0.185 - 0.027 = 0.02697$$

see above

2007, 3 → 2007, 4

$$\frac{12.63 \cdot 70}{1046.21}\left(\frac{75-70}{70}\right) + \frac{81 \cdot 2}{1046.21}\left(\frac{2-2}{2}\right) = 0.0604 + 0 = \frac{1109.37 - 1046.21}{1046.21} = 0.0604$$

7.5.1x (3) Digression: the numerical example ctd: 2009,1 → 2009,2

Average **prices** in 2008	of A ≈ **36.99**	of B ≈ **108.75**
quarter	**quantities**	
1	**70**	**4**
2	**90**	**2**
	weights	
quarter	**wA**	**wB**
1	**0.8551**	**0.1449**
2	**0.9387**	**0.0613**

2009,1 → 2009,2

$$g = \frac{\overline{V}_{y,y-1,q+1} - \overline{V}_{y,y-1,q}}{\overline{V}_{y,y-1,q}} = \frac{3546.16 - 3023.96}{3023.96} = 0.1727$$

$$= w_{Ayq}\left(\frac{q_{A,y,q+1} - q_{A,y,q}}{q_{A,y,q}}\right) + w_{Byq}\left(\frac{q_{B,y,q+1} - q_{B,y,q}}{q_{B,y,q}}\right)$$

$$= \frac{36.99 \cdot 70}{3023.96}\left(\frac{90-70}{70}\right) + \frac{108.75 \cdot 4}{3023.96}\left(\frac{2-4}{4}\right) =$$

$$\approx 0.24 - 0.07 = 0.17$$

Formulas for decomposing of growth rates (into contributions of certain aggregates to growth) are even more complicated

- for other comparisons (e.g. across years)
- or other linking techniques (that is for QO or OY).

Adding or chainlinking of (partial) growth rates does not make sense.

7.5.1x (4) Digression: contribution of net-exports to growth of GDP

net exports* = N = X - M

* balancing item B.11 (= external balance …)

The chain index deflation of balancing items (net export, inventories etc.) where varying signs may occur is not infrequently called in question

Y = GDP, F = final domestic expenditure: $Y = F + (X - M) = F + N$ and $\Delta Y = Y_t - Y_{t-1}$

This can be transformed to (see e.g. Kirchner)

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{\Delta N_t}{Y_{t-1}} + \frac{\Delta F_t / F_{t-1}}{1 + N_{t-1}/F_{t-1}}$$

growth rate of F (numerator $\Delta F_t / F_{t-1}$)

this part of denominator would vanish if $N_{t-1} = X_{t-1} - M_{t-1} = 0$

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{\Delta N_t}{Y_{t-1}} + \frac{\Delta F_t}{Y_{t-1}} = \frac{\Delta N_t}{N_{t-1}}\left(\frac{N_{t-1}}{Y_{t-1}}\right) + \frac{\Delta F_t}{F_{t-1}}\left(\frac{F_{t-1}}{Y_{t-1}}\right)$$

sum of "contributions"

weighted average of growth rates ([**variable**] weights in brackets); relevance of a changing structure of N relative to F

For interpretations in terms of "contributions" to growth **additivity is essential** and prerequisite

7.5.2 (1) Time series and comparisons: QO method

(24) year to year sequence of q=4 indices

$$I^{QO}_{y,q=4} = \frac{\sum_i \bar{p}_{i0} q_{i,1,q=4}}{\frac{1}{4}\sum_q \sum_i \bar{p}_{i0} q_{i,0,q}} \cdot \frac{\sum_i \bar{p}_{i1} q_{i,2,q=4}}{\sum_i \bar{p}_{i1} q_{i,1,q=4}} \cdot \ldots \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=4}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q=4}}$$

(25) sequence of quarterly indices

$$I^{QO}_{y,q=1} = I^{QO}_{y-1,q=4} \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=1}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q=4}} \qquad I^{QO}_{y,q=2} = I^{QO}_{y-1,q=4} \cdot \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q=2}}{\sum_i \bar{p}_{i,y-1} q_{i,y-1,q=4}}$$

(26) comparison D1 **(y, q) → (y, q-1)**

$$\frac{I^{QO}_{y,q}}{I^{QO}_{y,q-1}} = \frac{\bar{\bar{V}}_{y,y-1,q}}{\bar{\bar{V}}_{y,y-1,q-1}} = \frac{\sum_i \bar{p}_{i,y-1} q_{i,y,q}}{\sum_i \bar{p}_{i,y-1} q_{i,y,q-1}}$$

(26) QO = **(21)** AO — **pure** comparison!!

(27) comparison D2 **(y, q) → (y-1, q)**

$$\frac{I^{QO}_{y,q}}{I^{QO}_{y-1,q}} = \frac{\bar{\bar{V}}_{y,y-1,q}}{\bar{\bar{V}}_{y-1,y-2,q}} \cdot \frac{\bar{\bar{V}}_{y-1,y-2,q=4}}{\bar{\bar{V}}_{y-1,y-1,q=4}} = Q^{y,q}_{y-1,q} \div Q^{(*)y-1,y-1,4}_{y-1,y-2,4}$$

(28) comparison D3 **(y,q=4) → (y+1,q=1)**

$$\frac{I^{QO}_{y+1,q=1}}{I^{QO}_{y,q=4}} = L^{QO}_{y,q=4 \to y+1,q=1} = \frac{\bar{\bar{V}}_{y+1,y,q=1}}{\bar{\bar{V}}_{y,y,q=4}} = D$$

pure quantity-comparison, prices of y

see eq. 23a

It can easily be verified that due to **(26)** growth factors $I_{y,2}/I_{y,1}$ or $I_{y,3}/I_{y,2}$ and $I_{y,4}/I_{y,3}$ of both methods, the QO and AO method are in fact the same

7.5.2 (2) Alternative presentation of eqs. (24), (25) QO method

y	q = 1	…	q = 4	annual **QO** index (y)
0*	$I^{QO}_{0,1} = \frac{\overline{V}_{0,0,q=1}}{\frac{1}{4}W_0}$		$I^{QO}_{0,4} = \frac{\overline{V}_{0,0,q=4}}{\frac{1}{4}W_0}$	$\frac{\sum_q \overline{V}_{0,0,q}}{W_0} = \frac{1}{4}\sum_q I^{QO}_{0,q} = I^{QO}_0 = 1$
1	$I^{QO}_{1,1} = I^{QO}_{0,4}\frac{\overline{V}_{1,0,q=1}}{\overline{V}_{0.0.4}}$		$I^{QO}_{1,4} = I^{QO}_{0,4}\frac{\overline{V}_{1,0,q=4}}{\overline{V}_{0,0,q=4}}$	$I^{QO}_1 = I^{QO}_{0,4}\frac{\frac{1}{4}\sum_q \overline{V}_{1,0,q}}{\overline{V}_{0,0,q=4}} = \frac{\sum_q \overline{V}_{1,0,q}}{\frac{1}{4}W_0}$
2	$I^{QO}_{2,1} = I^{QO}_{1,4}\frac{\overline{V}_{2,1,q=1}}{\overline{V}_{1.1.q=4}}$		$I^{QO}_{2,4} = I^{QO}_{1,4}\frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1.1.q=4}}$	$I^{QO}_2 = I^{QO}_{1,4}\frac{\frac{1}{4}\sum_q \overline{V}_{2,1,q}}{\overline{V}_{1,1,q=4}}$
3	$I^{QO}_{3,1} = I^{QO}_{2,4}\frac{\overline{V}_{3,2,q=1}}{\overline{V}_{2,2,q=.4}}$		$I^{QO}_{3,4} = I^{QO}_{2,4}\frac{\overline{V}_{3,2,q=4}}{\overline{V}_{2,2,.q=4}}$	$I^{QO}_3 = I^{QO}_{2,4}\frac{\frac{1}{4}\sum_q \overline{V}_{3,2,q}}{\overline{V}_{2,2,q=4}}$

* same result as AO method

chain index for q = 4

$$I^{QO}_{y,4} = \frac{\overline{V}_{1,0,q}}{\frac{1}{4}W_0}\frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1,1,q=4}}\frac{\overline{V}_{3,2,q=4}}{\overline{V}_{2,2,q=4}}\cdots\frac{\overline{V}_{y,y-1,q=4}}{\overline{V}_{y-1,y-1,q=4}}$$

to compare **(27)** for QO to **(22)** for AO (same quarter different years)

27 $$\frac{I^{QO}_{y,q}}{I^{QO}_{y-1,q}} = \frac{\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-2,q}} \div \frac{\overline{V}_{y-1,y-1,q=4}}{\overline{V}_{y-1,y-2,q=4}} \cdot = Q^{y,q}_{y-1,q} \div Q^{(*)y-1,y-1,4}_{y-1,y-2,4}$$

22 $$\frac{I^{AO}_{y,q}}{I^{AO}_{y-1,q}} = Q^{y,q}_{y-1,q} \div \frac{\sum_q \overline{V}_{y-1,y-1,q}}{\sum_q \overline{V}_{y-1,y-2,q}} = Q^{y,q}_{y-1,q} \div A^{y-1,y-1}_{y-1,y-2}$$

Note:
27 and 22 differ only with respect to Q* (referring to q = 4) or the Paasche price indices A (referring to a year), respectively.

Whenever the fourth quarter is representative of the whole year, that is A ≈ Q* then also OQ ≈ AO. Comparison is **biased**

However, the comparison **D3** **(y,q=4 → y+1,q=1)**

28 $$\frac{I^{QO}_{y+1,q=1}}{I^{QO}_{y,q=4}}$$ turns out to be a **pure** quantity comparison

$A^{y-1,y-1}_{y-1,y-2}$ is lagging one period behind $A^{y,y}_{y,y-1}$ in (**23**) slide 59 to compare **(28)** for QO to **(23)** for AO (comparison D3)

28 $$\frac{I^{QO}_{y+1,q=1}}{I^{QO}_{y,q=4}} = \frac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y,q=4}}$$

this is exactly D of eq. **23a**

$$D = \sum_i \overline{p}_{iy} q_{i,y+1,q=1} \Big/ \sum_i \overline{p}_{iy} q_{i,y,q=4}$$

hence: **pure** quantity comparison

However with the AO technique we get

23 $$\frac{I^{AO}_{y+1,q=1}}{I^{AO}_{y,q=4}} = \frac{\overline{V}_{y+1,y,q=1}}{\overline{V}_{y,y-1,q=4}} \div \frac{\sum_q \overline{V}_{y,y,q}}{\sum_q \overline{V}_{y,y-1,q}} = Q^{y+1,1}_{y,4} \div A^{y,y}_{y,y-1}$$

The first factor Q is unequal to D, and A is again a Paasche price index.

Note $$A^{y,y}_{y,y-1} = \frac{1}{A^{y,y-1}_{y,y}}$$

7.5.3 (1) Time series and comparisons: OY method (quarter successive years)

(29) sequence of indices (quarter q over the years)	$$I^{OY}_{y,q} = \frac{\overline{V}_{1,0,q}}{\frac{1}{4}W_0} \cdot \frac{\overline{V}_{2,1,q}}{\overline{\overline{V}}_{1,1,q}} \cdot \frac{\overline{V}_{3,2,q}}{\overline{\overline{V}}_{2,2,q}} \cdot \ldots \cdot \frac{\overline{V}_{y,y-1,q}}{\overline{\overline{V}}_{y-1,y-1,q}}$$ **note**: prices of 0,1,…,y-1, quantities of years 0, 1, …,y
(30) sequence of quarterly indices (year y)*	$$I^{OY}_{y,q=1} = I^{OY}_{y-1,q=1} \frac{\overline{V}_{y,y-1,q=1}}{\overline{\overline{V}}_{y-1,y-1,q=1}} = I^{OY}_{y-1,q=1} L_{y,1}$$ L = link see **(15)**
(31) comparison D1 **(y,q) → (y,q-1)**	$$\frac{I^{OY}_{y,q=2}}{I^{OY}_{y,q=1}} = \frac{I^{OY}_{y-1,q=2} L_{y,2}}{I^{OY}_{y-1,q=1} L_{y,1}}$$ not meaningful; $L_{y,q} = L_{y-1,q \rightarrow y,q}$
(32) comparison D2 **(y,q) → (y-1,q)**	$$\frac{I^{OY}_{y,q}}{I^{OY}_{y-1,q}} = L^{OY}_{y-1,q \rightarrow y,q} = \frac{\overline{V}_{y.y-1,q}}{\overline{\overline{V}}_{y-1.y-1,q}}$$ = **(17)** same prices in numerator and denominator (**pure comparison**)
(33) comparison D3 **(y,q=4) → (y+1,q=1)**	$$\frac{I^{OY}_{y+1,q=1}}{I^{OY}_{y,q=4}} = \frac{I^{OY}_{y,q=1} L_{y+1,1}}{I^{OY}_{y,q=4}} = \frac{I^{OY}_{y,q=1}}{I^{OY}_{y,q=4}} \frac{\overline{V}_{y+1,y,q=1}}{\overline{\overline{V}}_{y,y,q=1}}$$ **biased**

* for q=4 QO is the same as OY

7.5.3 (2) Alternative presentation of eqs. (29), (30) OY method

y	q = 1	…	q = 4	annual **OY** index (y)
0*	$I_{0,1}^{OY} = \frac{\overline{\overline{V}}_{0,0,q=1}}{\frac{1}{4}W_0}$		$I_{0,4}^{OY} = \frac{\overline{\overline{V}}_{0,0,q=4}}{\frac{1}{4}W_0}$	$I_0^{OY} = \frac{1}{4}\sum_q I_{0,q}^{OY} = 1$
1*	$I_{1,1}^{OY} = I_{0,1}^{OY}\frac{\overline{\overline{V}}_{1,0,q=1}}{\overline{\overline{V}}_{0,0,q=1}} = \frac{\overline{\overline{V}}_{1,0,q=1}}{\frac{1}{4}W_0}$		$I_{1,4}^{OY} = \frac{\overline{\overline{V}}_{1,0,q=4}}{\frac{1}{4}W_0}$ **	$I_1^{OY} = \frac{1}{4}\sum_q I_{1,q}^{OY}$
2	$I_{2,1}^{OY} = I_{1,1}^{OY}\frac{\overline{\overline{V}}_{2,1,q=1}}{\overline{\overline{V}}_{1,1,q=1}}$		$I_{2,4}^{OY} = I_{1,4}^{OY}\frac{\overline{\overline{V}}_{2,1,q=4}}{\overline{\overline{V}}_{1,1,q=4}}$ **	general $I_y^{OY} = \frac{1}{4}\sum_q I_{y,q}^{OY}$

* same result as the AO and QO method

** same result as QO method

The four chain indices q = 1, …, 4

(29) $$I_{y,q}^{OY} = \frac{\overline{\overline{V}}_{1,0,q}}{\frac{1}{4}W_0}\cdot\frac{\overline{\overline{V}}_{2,1,q}}{\overline{\overline{V}}_{1,1,q}}\cdot\frac{\overline{\overline{V}}_{3,2,q}}{\overline{\overline{V}}_{2,2,q}}\cdot\ldots\cdot\frac{\overline{\overline{V}}_{y,y-1,q}}{\overline{\overline{V}}_{y-1,y-1,q}}$$

7.5.4 (1) Chaining: which indices are chained indices and which are derived from them?

AO: annual indices

$$I_y^{AO} = \frac{\sum_q \overline{V}_{1,0,q}}{W_0} \frac{\sum_q \overline{V}_{2,1,q}}{W_1} \frac{\sum_q \overline{V}_{3,2,q}}{W_2} \cdots \frac{\sum_q \overline{V}_{y,y-1,q}}{W_{y-1}}$$

derived **(7)**, **(8)** $I_{y,q=1}^{AO} = I_{y-1}^{AO} \frac{\overline{V}_{y,y-1,q=1}}{\frac{1}{4} W_{y-1}}$, $I_{y,q=2}^{AO} = I_{y-1}^{AO} \frac{\overline{V}_{y,y-1,q=2}}{\frac{1}{4} W_{y-1}}$ etc.

QO: indices for q = 4 over the years

$$I_{y,4}^{QO} = \frac{\overline{V}_{1,0,q=4}}{\frac{1}{4} W_0} \frac{\overline{V}_{2,1,q=4}}{\overline{V}_{1,1,q=4}} \frac{\overline{V}_{3,2,q=4}}{\overline{V}_{2,2,q=4}} \cdots \frac{\overline{V}_{y,y-1,q=4}}{\overline{V}_{y-1,y-1,q=4}}$$

derived: quarters q=1, 2, 3 **(11)**, **(12)** annual indices (average of quarterly indices) **(13)**

OY: year to year indices for quarter q = 1, …,4 **(29)**

$$I_{y,q}^{QO} = \frac{\overline{V}_{1,0,q}}{\frac{1}{4} W_0} \frac{\overline{V}_{2,1,q}}{\overline{V}_{1,1,q}} \frac{\overline{V}_{3,2,q}}{\overline{V}_{2,2,q}} \cdots \frac{\overline{V}_{y,y-1,q}}{\overline{V}_{y-1,y-1,q}}$$

$$I_{y,q=4}^{OY} = I_{y,4}^{QO}$$

derived annual index **(17)**, **(18)** $I_y^{OY} = \frac{1}{4} \sum_q I_{y,q}^{OY}$

7.5.4 (2) Chaining and comparison of the annual indices (1)

y	CP index*	AO	QO	OY
05	151.30	151.30	151.30	151.30
06	201.07	201.07	200.49	200.65
07	232.58	232.58	231.06	239.33
08	224.44	224.44	218.96	227.06

* at constant average prices of 2005

Sequence of CP indices (direct indices): $I_{05,06}$, $I_{05,07}$, …

Products: Annual index formulas (**chain** index formulas **AO, QO, OY**): first factor $I_{05,06}$ (base 05, y = 06);first two factors $I_{05,07;}$ first three $I_{05,07}$ etc

1) Sequence of **direct CP indices**
Laspeyres volume indices

$$I_{01}^{CP} = \frac{\sum\sum \bar{p}_{i0} q_{i1q}}{\sum\sum \bar{p}_{i0} q_{i0q}} \qquad I_{02}^{CP} = \frac{\sum\sum \bar{p}_{i0} q_{i2q}}{\sum\sum \bar{p}_{i0} q_{i0q}}$$

or equivalently

$$I_{01}^{CP} = \frac{\overline{V}_{1,0}}{\overline{V}_{0,0}}, \quad \overline{V}_{0,0} = \sum\sum \bar{p}_{i0} q_{i0q} \qquad I_{02}^{CP} = \frac{\overline{V}_{2,0}}{\overline{V}_{0,0}}$$

2) **AO** annual indices **chain** index **(19)**

$$I_{y}^{AO} == \frac{\sum\sum \bar{p}_{i0} q_{i1q}}{\sum\sum \bar{p}_{i0} q_{i0q}} \cdot \frac{\sum\sum \bar{p}_{i1} q_{i2q}}{\sum\sum \bar{p}_{i1} q_{i1q}} \cdot \ldots \cdot \frac{\sum\sum \bar{p}_{i,y-1} q_{i,y,q}}{\sum\sum \bar{p}_{i,y-1} q_{i,y-1,q}}$$

or equivalently

$$I_{y}^{AO} == \frac{\overline{V}_{1,0}}{\overline{V}_{0,0}} \cdot \frac{\overline{V}_{2,1}}{\overline{V}_{1,1}} \cdot \ldots \cdot \frac{\overline{V}_{y,y-1}}{\overline{V}_{y-1,y-1}}$$

follows the rationale of chain price indices

7.5.4 (3) Comparison of the annual indices (2)

3) **QO** annual indices **chain** index **(24)**

the annual index is not a chain index (only $I_{y,q=4}$ is a chain index) **but an unweighted arithmetic mean of the four quarterly indices**

when the fourth quarter is representative of the whole year

this applies also to

or $$\overline{V}_{y-1,y-1,q=4} \approx \frac{1}{4}\overline{V}_{y-1,y-1}$$ then OQ ≈ AO

4) **OY** annual indices

Although some annual indices are derived from quarterly indices this does not mean that in these cases QNA is consistent with ANA (aggregated QNA volumes equal directly derived ANA volumes)

Experience has shown that QO is the most problematic method regarding non-additivity in time and inconsistency between QNA and ANA (that is QO will violate "time consistency" in the most pronounced manner)

7.6 (1) Methods and their evaluation

advantages are highlighted

	Annual overlap (AO)	Quarterly overlap (QO)	over the year (OY)
Comparisons **D1** (y, q) → (y, q+1)	pure comparison* **unbiased** (21) same prices depending on quantities only	**unbiased** (26) = (21)	not meaningful (31)
D2 (y, q) → (y+1, q)	**biased** (22, 27) changing price weights		**unbiased** (32)
D3 (y, 4) → (y+1, 1)	biased (23)**	unbiased (28)	biased (33)
AC additivity over aggregates	as a rule additivity only in the base (= reference) year (and the following year); all other years non-additive; the discrepancy can well be substantial (significant)		

* volumes based on the same prices in numerator and denominator

** that is there is a break between 4th quarter of one year and 1st of following year; unbiased would be eq. 23a

7.6 (2) Methods and their evaluation

	Annual overlap (AO)	Quarterly overlap (QO)	over the year (OY)
AC compa-rability + de-composition of **growth rates**	despite same price weights growth rates yq/y,q-1 (be-tween successive quarters)* *not* easily decomposable	growth rates except between y,q=4 and y+1,q=1 influenced by different prices	growth rates y,q vs. y-1,q depend only on changes in the quantities
AT ** time aggregation	chained QNA figures sum up to ANA results	criterion not (or only approximately for OY) met; need for additional bench-marking	
Main **advantage**	Time consistency (AT), annual indices (y →y+1) undistorted	quarter on quarter compar. undistorted for all quarters of y	re-valuation necessary for the all quarters of each year
Main **dis-advantage**	Discontinuity y,4 → y+1,1 and in general in q=1 growth rates (difference between AO and QO indication of "drift" (time-inconsistency of QO)	no time consistency AT, remediable by benchmarking [con-strained QO]	structural break in any y,q →y,q+1; basically four sepa-rate time series

* other growth rates will in general be influenced by a change in the price weights and thus even less comparable into "contributions"

** also known as "time consistency"

	Annual overlap (AO)	Quarterly overlap (QO)	over the year (OY)
quarterly **growth rates y,q → y,q+1**	Identical growth for all quarters other than across year joins. As QO is not time consistent (has a "drift") the difference between y,4 and y+1,1 AO and QO growth rate accounts for the drift		dicontin. in the growth rates; index for q = 4 is equal to the QO q=4 index*
Ease of computa-tion	no need to re-value any quarters at average prices of the current year	re-valuation is nec-essary for the fourth quarter only	re-valuation necessary for the all quarters of each year
Usage of the method	majority of EU Member States (for more detail see Kuhnert)	recommended by Eurostat, USA, UK, WIFO (in A)	NL (for unadjusted, AO for adjusted)

In addition to time consistency no discontinuities between successive quarters is desirable because the linking technique should allow growth to be estimated over varying period lengths

* time consistency (AT) is approximately fulfilled because contributions of the quarters to the drift tend to counterbalance each other

7.6 (3) Merits and demerits of the methods

Other observations, some empirical findings and more general statements

AO: breaks in q=1 of y+1	Scheiblecker with ref. to Bikker and own Austrian empirical results: AO is equivalent to QO with a built-in pro-rata benchmarking which is the reason for the break at the beginning of a year; They (asa well as IMF) recommend a "bench-marked QO"* (or "restricted QO) method and/or smoothing of the stepped line of AO figures
QO: QNA-ANA gap	Scheiblecker found that the differences between accumulated QNA and independently derived ANA were the largest in the case of QO
QO: growth rates	Growth rates in y-1,q → y,q (previous year) comparison are higher in QO than with the AO technique (Nierhaus)

* method of Denton: minimizing the relative difference of the relative adjustments of two neighbouring quarters

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