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Problems with Chain Indices (IV)

(Seasonal adjustment, Time aggregation, and some additional problems)

Course delivered at the European Central Bank

8. Interaction of seasonal adjustment and chain-linking of quarterly volume measures
   - 8.1 Terminology and problem
   - 8.2 Recommendations and findings
   - 8.3 A simulation
9. Other methodological problems with chain indices
   - 9.1 Eliminating chain drift using methods of international comparisons (consistent time aggregation with chain indices)
   - 9.2 Critique of the GEKS method of time aggregation
   - 9.3 Digression: Retrospective Pseudo-Fisher Indices (PFI)
   - 9.4 Optimal interval for linking
   - 9.5 A theory on similarity of price structures (vectors)
10. Collateral damages of the chain-o-mania
    - 10.1 Irrelevance of the axiomatic approach

8.1 (1) Chain-linking and seasonal adjustment: Terminology and problems

"volume *measure*" = index, or volumes as monetary values, or growth rates

order of carrying out the operations of

C chain-linking

which technique to be chose? Breaks due to technique?

S seasonal ad- justment*

when? C-S or S-C
at which level? S is usual- ly carried out at a higher level of aggregation

aggregation (**A**), benchmarking (**B**)

more general: observing **restrictions**, such as

1) time consistency by "***benchmarking***"
2) ***aggregation*** over components of an aggregate
3) ***reconciliation*** of estimations e.g. pro- duction vs expenditure approach in NA

aggregates: absolute volumes in t-1, t, t+1

chain-linking →
← unchaining

chain index (links and chain) $I_{t-1}I_t$ …

they do not constitute a time series

* and calendar (working-day) adjustment

8.1 (2) Chain-linking and seasonal adjustment: Terminology and problems

direct and indirect seasonal adjustment

indirect = adjust components then aggregate
direct = adjust the total aggregate

The choice between indirect and direct requires a case-by-case decision. Direct may be superior because adjustment is better done on a higher level of aggregation.
Discrepancies between direct vs. indirect ⇒

Procedure	possible effect	things to do
Benchmarking (or: "scaling")	may induce an artificial seasonal pattern	check for seasonality after benchmarking
(Seasonal) Adjustment S	time consistency of ANA measures may not be preserved after S	check adjusted series for time consistency; time consistency may be "forced" as opposed to additivity
indirect adjustment	a volume measure which is derived from component series or as residual* may have zero or negative values although each of its components is strictly positive * e.g. net exports, inventories	

8.1 (3) Some observations on chain-linking and seasonal adjustment

Procedure	possible effect of seasonal adjustment	things to do
Seasonality and "drift"* (Kenny)	removing seasonal component may or may not reduce drift; the situation differs widely across industries; most important: correlation between price and quantity movement	carry out seasonal adjustment at the earliest possible stage
interaction between the operations	"It seems unlikely that any general algebraic solution to these problems can be found, so simulation is likely to be the only viable approach."	Use real data, as "the construction of data dictates the solution"

* "drift" in the sense of discrepancy between q = 1 estimate of AO as opposed to QO

Linking technique

1. OY as chain-linking method not recommended
2. Linking technique has impact on
   - structural breaks
   - seasonality, identifying models such as ARIMA
   - detection of outliers
3. AO does in general not require benchmarking

Sequence of operations

1. Seasonal adjustment S should be carried out *after* chain-linking C,
2. possibly followed by bench-marking (B) of the adjusted series, that is the order of procedures should be*

   C → S → B (rather than S-C)

* Benchmarking may produce or distort sea-sonality and therefore should not be made before adjustment. If aggregation (A) is needed

C → A → S → B

Recommendations concerning restrictions

1. Additivity

a) It is not recommendable to require forcing additivity of components and their respective aggregate after chain-linking (as to the residuals ⇒)
("no correction should be made to remove non-additivity")

b) produce a set of adjusted volume measures which are additive when ex-pressed in prices of the previous year

2. Time consistency

adjusted (S) *quarterly* measures should be forced to be equal to *non adjusted annual* measures

3. other restrictions

After making corrections for complying with them (e.g. after a reconciliation process) test for seasonality.
Calendar adjusted quarterly data should not be benchmarked to unadjusted annual data.

discrepancies due to non-additivity

1. allocate discrepancies between non-adjusted totals and components in previous year prices to a ***series of discrepancies*** and transmit to Eurostat or
2. allocate it to a series of a residual character (e.g. changes in inventories) or
3. distribute them among the components (!!!)

It is not clear what Eurostat is going to do with the series of discrepancies

The following **section 8.3** was meant to summarize some experiences (with real data) with performing adjustment and linking in a different order of operation. I therefore presented in this part as a first step some empirical data (time series) although the figures might perhaps better fit to part III of this presentation.

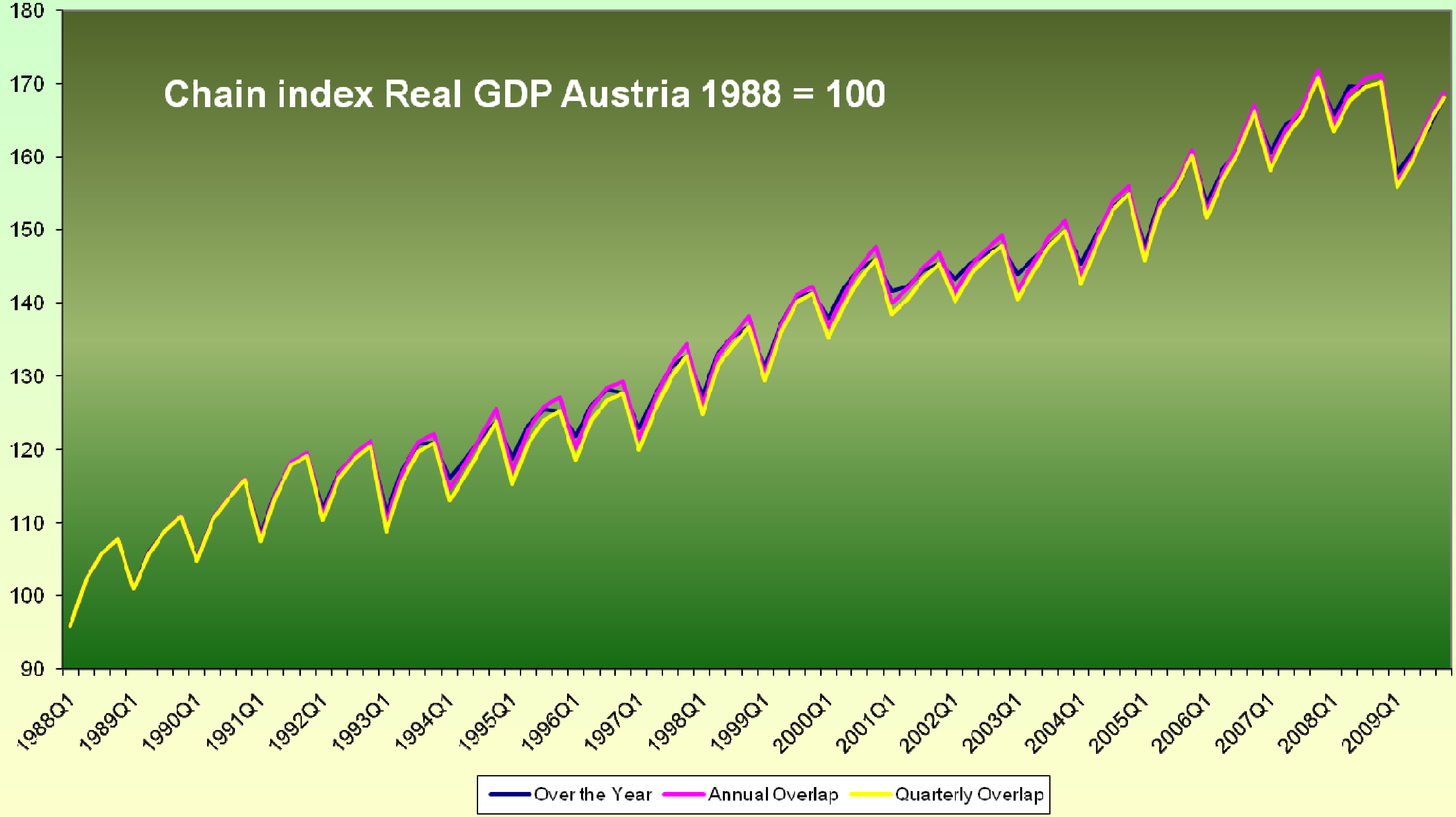
8.3 (1) Corrigendum (of the version delivered at the ECB, January 2010)

The German Bundesbank asked me to make public her following statement referring to my original presentation of the course as given on Jan. 19th 2010

- The calculations for German chain-linked GDP shown in the course given at the ECB were erroneous. In fact, data for Germany do not allow the calculation of a linking technique other than Annual Overlap (volumes in current year average prices are not published).
- Moreover, prior to the introduction of chain indices, both the German Federal Statistical Office and the Deutsche Bundesbank conducted several control calculations. These revealed that generally the results from Annual Overlap and Quarterly Overlap are close together.
- However, for Austria (unadjusted) data are available that allow the calculation of all three linking techniques (thanks to Marcus Scheiblecker from WIFO).* The results show only minor differences in the growth rates of the Annual Overlap and Quarterly Overlap techniques - most importantly, both series are not drifting apart.
- Note that in the practice of National Accounts the results from the Quarterly Overlap technique are benchmarked against those derived from the Annual Overlap technique. This is done in order to ensure that the quarterly results match the annual ones.

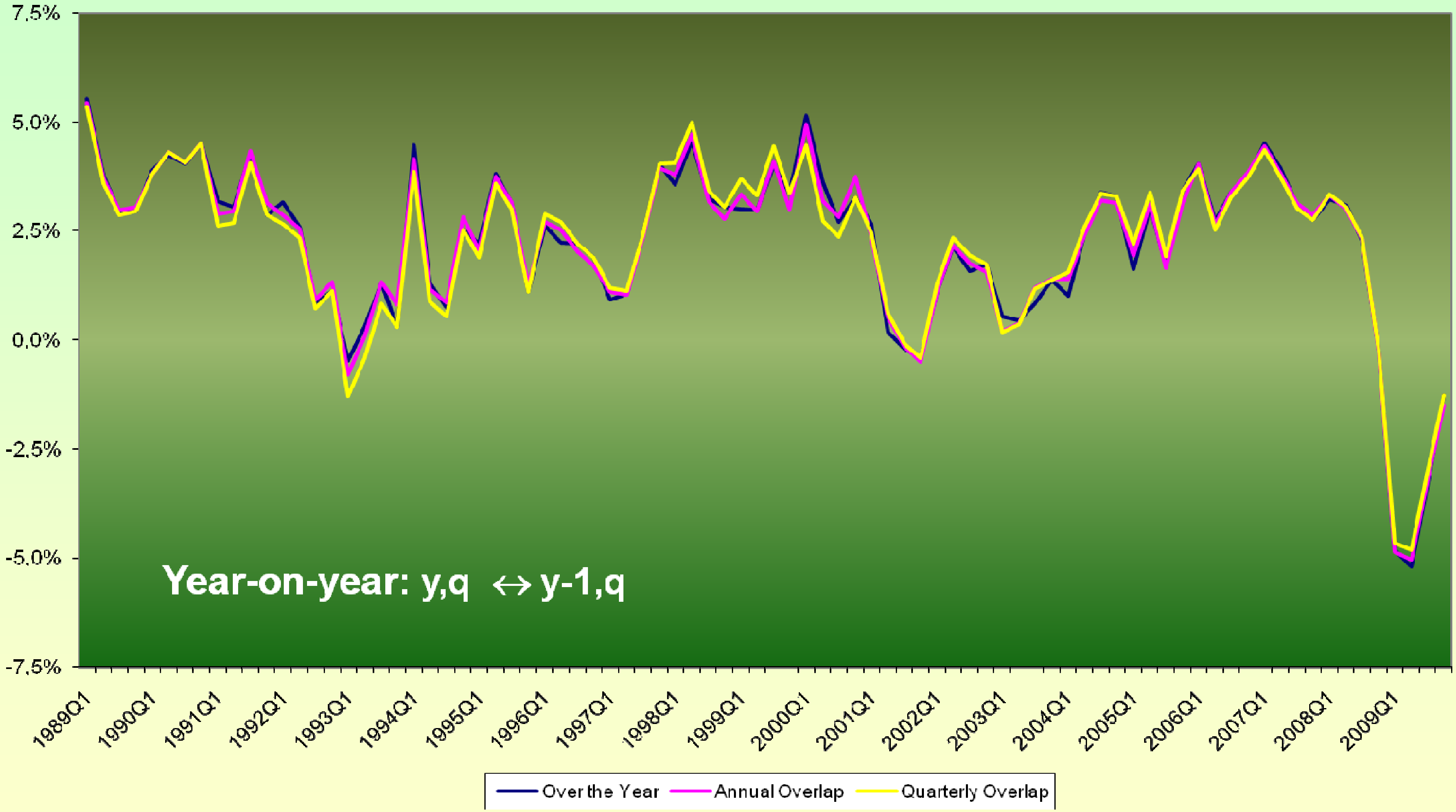
* see the following slides

8.3 (2) Austrian GDP time series: the volume index (owing to Marcus Scheiblecker)



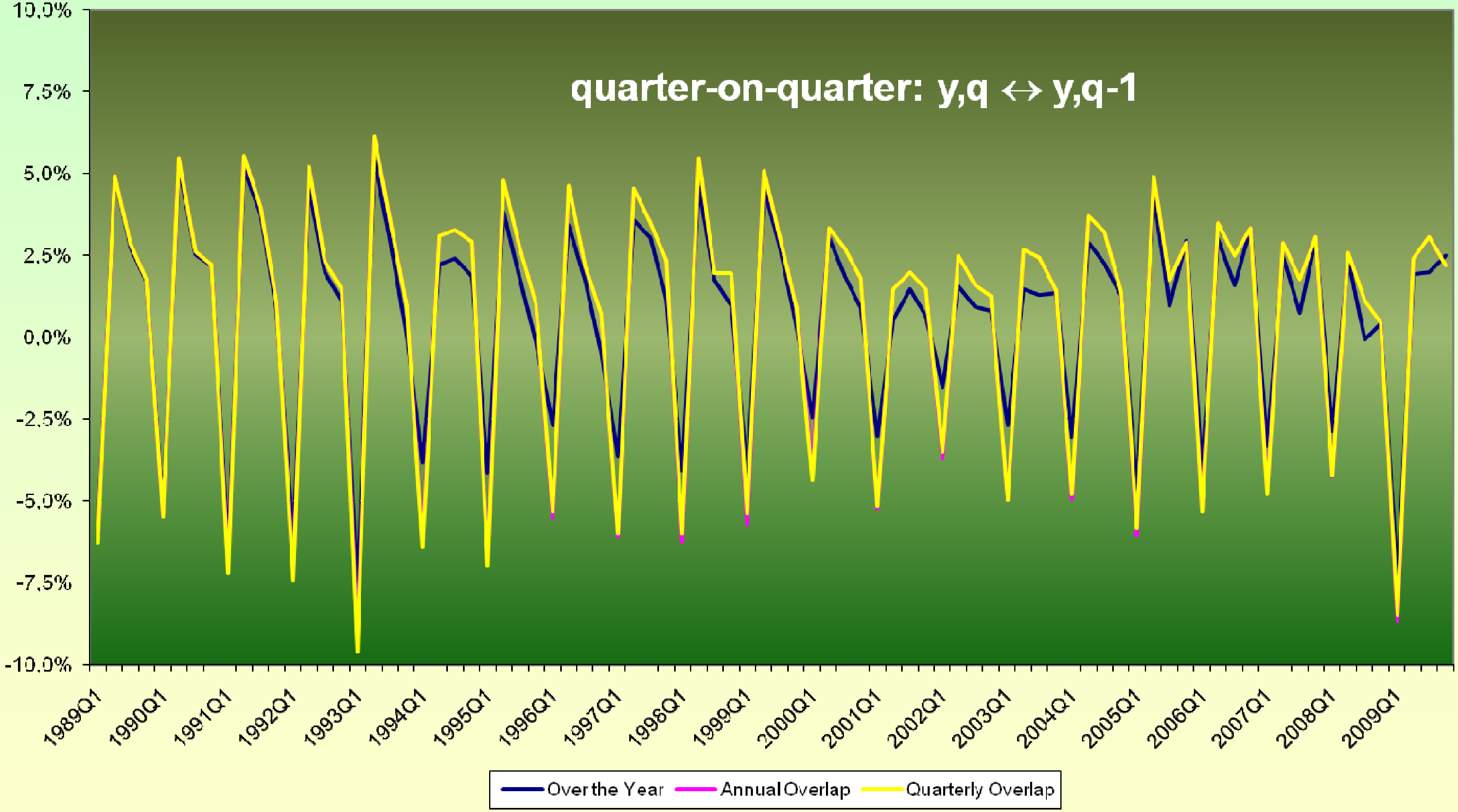
8.3 (3) Austrian GDP time series: growth rates (owing to Marcus Scheiblecker)

Note that growth rates of AO and QO must be the same except for the first quarter of a year

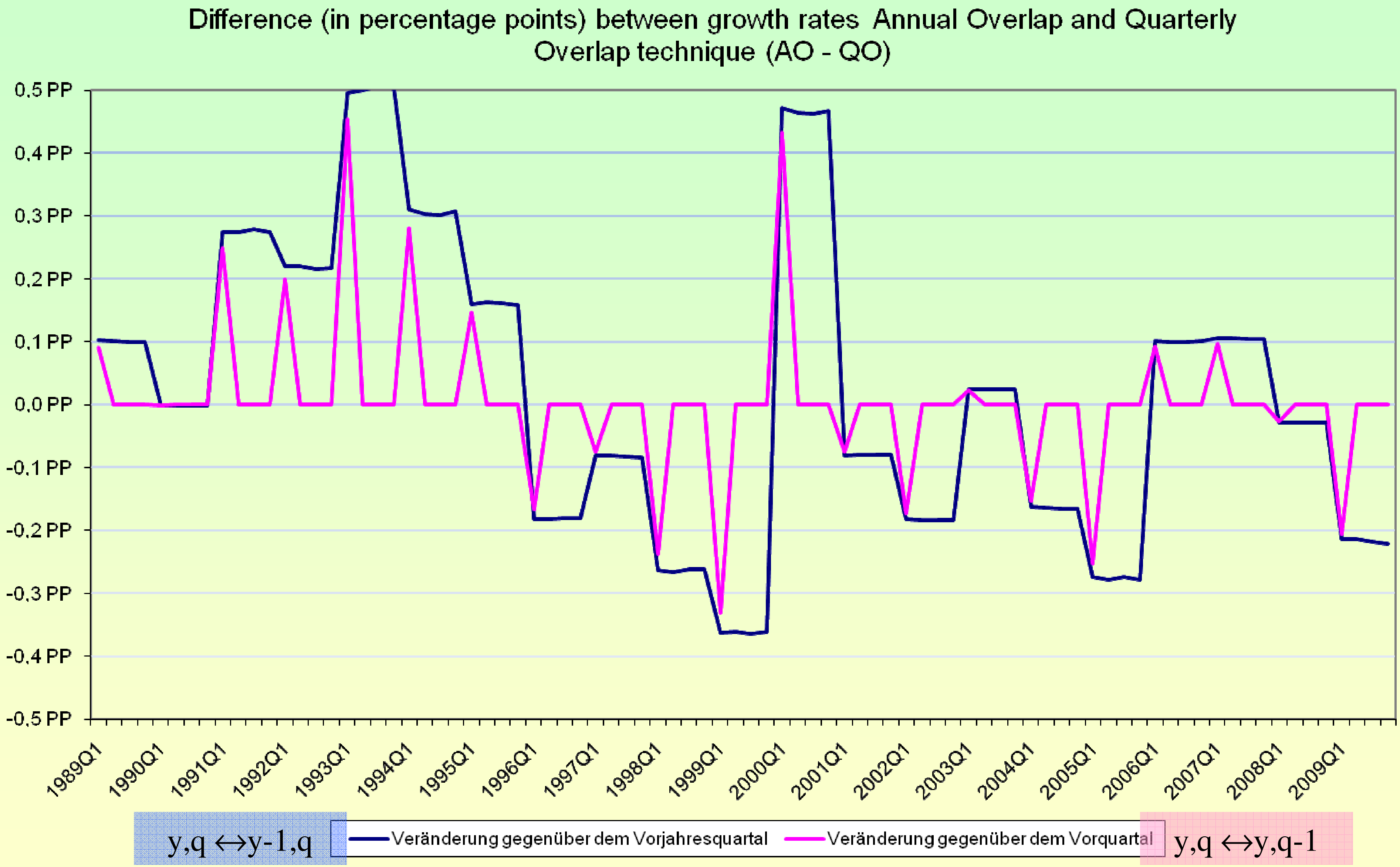


8.3 (4) Austrian GDP time series: growth rates (owing to Marcus Scheiblecker)

Interestingly OY is not that much different from AO and QO as expected



8.3 (5) Austrian GDP time series (owing to Markus Scheiblecker)



9.1 (1) Eliminating chain drift using methods of international comparisons

IFD = Lorraine Ivancic, Kevin J. Fox, W. Erwin Diewert, Scanner Data, Time Aggregation and the Construction of Price Indexes, May 2009

The notion of "chain **drift**"

1. here (IFD): multiperiod identity is failed (more general: no transitivity)
2. drift function:

$$\overline{P}_{0t}^{LC} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \frac{\sum p_2 q_1}{\sum p_1 q_1} \cdots \frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \neq P_{0t}^{L} = \frac{\sum p_t q_0}{\sum p_0 q_0}$$

3. in the framework of QNA linking techniques: AO and QO estimate for the first quarter of a year different.

Relevance: increasingly relevant when using high-frequent scanner data

Method: Methods designed for making transitive international comparisons such that $P_{AC} = P_{AB} P_{BC}$ appear useful

substitute as follows: replace countries A, B, C by periods t-1, t, t+1

bilateral problems

- asymmetric (unbalanced) P^L, P^P
- symmetric (balanced) P^F, P^{DRO}, P^T
  - Country reversibility test (CRT) failed
  - CRT satisfied: P^F

multilateral problems

- nicht transitiv (P^F)
- transitiv
  - weak transitivity: CCM = central country method
  - strict transitivity (= base land invariance)

bloc methods: GK (Geary Khamis), averaging methods: **EKS** (Eltetö - Köves – Szulc) oder CCD (Caves-Christensen-Diewert)

GEKS method, however, fairly complicated

$$P_{AC}^{EKS} = \sqrt[3]{\left(P_{AC}^{F}\right)^{2} P_{AB}^{F} P_{BC}^{F}}$$

$$P_{AC}^{EKS} = \sqrt[4]{\left(P_{AC}^{F}\right)^{2} P_{AB}^{F} P_{BC}^{F} P_{AD}^{F} P_{DC}^{F}}$$

IFD Terminology
GEKS instead of EKS (= Gini-EKS)
GEKS denotes also Generalized EKS

9.1 (3) More details and formal relations concerning the (G)EKS parities

$$P_{kj}^{EKS} = \frac{P_j}{P_k} = \left(\frac{P_{1j}^F}{P_{1k}^F} \frac{P_{2j}^F}{P_{2k}^F} \cdots \frac{P_{mj}^F}{P_{mk}^F} \right)^{1/m} = \left(\prod_l \frac{P_{lj}^F}{P_{lk}^F} \right)^{1/m}$$

$$P_{kj}^{EKS} = \frac{P_j}{P_k} = \left(\frac{P_{k1}^F}{P_{j1}^F} \frac{P_{k2}^F}{P_{j2}^F} \cdots \frac{P_{km}^F}{P_{jm}^F} \right)^{1/m} = \frac{\sqrt[m]{\prod_l P_{kl}^F}}{\sqrt[m]{\prod_l P_{jl}^F}}$$

$$P_{AC}^{EKS} = \left(\frac{P_{AC}^F}{P_{AA}^F} \frac{P_{BC}^F}{P_{BA}^F} \frac{P_{CC}^F}{P_{CA}^F} \right)^{1/3} = \left(\frac{P_{AA}^F}{P_{CA}^F} \frac{P_{AB}^F}{P_{CB}^F} \frac{P_{AC}^F}{P_{CC}^F} \right)^{1/3} = \sqrt[3]{\left(P_{AC}^F\right)^2 P_{AB}^F P_{BC}^F}$$

or as a product

$$\boxed{P_{kj}^{EKS} = \sqrt[m]{\prod_l P_{kl}^F \prod_l P_{lj}^F}}$$

Derivation of GEKS*
v.d.L. (2007), p. 555f)

Two equivalent interpretations

$$P_{AC}^{EKS} = \left[\underbrace{P_{AA}^F P_{AC}^F}_{i=A} \underbrace{P_{AB}^F P_{BC}^F}_{i=B} \underbrace{P_{AC}^F P_{CC}^F}_{i=C} \right]^{1/3} = \sqrt[3]{\left(P_{AC}^F\right)^2 P_{AB}^F P_{BC}^F}$$

$$P_{AC}^{EKS} = \left[\left(P_{AA}^F P_{AB}^F P_{AC}^F\right)\left(P_{AC}^F P_{BC}^F P_{CC}^F\right) \right]^{1/3} = \sqrt[3]{\left(P_{AC}^F\right)^2 P_{AB}^F P_{BC}^F}$$

so formulas quickly (as m increases) **get quite complicated**

9.2 (1) Critique of using EKS in order to avoid "chain drift": complicated formulas

general formula $P_{0T}^{EKS} = \left[\left(P_{0T}^{F}\right)^2 \prod_{t \neq 0} P_{0t}^{F} \prod_{t \neq T} P_{tT}^{F} \right]^{1/m}$

1) P_{st} depends on the number m of periods

"…when a new period of data becomes available all of the previous period parities must be recomputed" (IFD, p. 22) → avoid with RWGEKS

periods	P_{02}	direct Fisher	chain Fisher
0, 1, 2	$\sqrt[3]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{12}^{F}}$	P_{02}^{F}	$P_{01}^{F} P_{12}^{F}$
0, 1, 2, 3	$\sqrt[4]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{12}^{F} P_{03}^{F} P_{32}^{F}}$	P_{02}^{F}	$P_{01}^{F} P_{12}^{F}$
0, 1, 2, 3, 4	$\sqrt[5]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{12}^{F} P_{03}^{F} P_{32}^{F} P_{04}^{F} P_{42}^{F}}$	P_{02}^{F}	$P_{01}^{F} P_{12}^{F}$
0, 1, 2, 3, 4, 5	$P_{02(6)}^{EKS} = \sqrt[6]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{13}^{F} P_{03}^{F} P_{32}^{F} P_{04}^{F} P_{42}^{F} P_{05}^{F} P_{52}^{F}}$	P_{02}^{F}	$P_{01}^{F} P_{12}^{F}$

which is the "target" (= correct and drift-free) index?

9.2 (2) Complicated formulas spelled out in detail and their interpretation

Direct Fisher	P_{02}^{F}	$\sqrt{\frac{\mathbf{p}_2'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_0'\mathbf{q}_2}}$
Chain Fisher	$P_{01}^{F}P_{12}^{F}$	$\sqrt{\frac{\mathbf{p}_1'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_1'\mathbf{q}_1}{\mathbf{p}_0'\mathbf{q}_1}\frac{\mathbf{p}_2'\mathbf{q}_1}{\mathbf{p}_1'\mathbf{q}_1}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_1'\mathbf{q}_2}}$

m = 6 periods means no less than 2(12-3) = 18 ratios and 36 aggregates influencing the result

$$P_{02(6)}^{EKS} = \sqrt[6]{\left(P_{02}^{F}\right)^2 P_{01}^{F}P_{12}^{F}P_{03}^{F}P_{32}^{F}P_{04}^{F}P_{42}^{F}P_{05}^{F}P_{52}^{F}}$$

$$\sqrt[6]{\frac{\mathbf{p}_2'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_0'\mathbf{q}_2}\sqrt{\frac{\mathbf{p}_1'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_1'\mathbf{q}_1}{\mathbf{p}_0'\mathbf{q}_1}\frac{\mathbf{p}_2'\mathbf{q}_1}{\mathbf{p}_1'\mathbf{q}_1}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_1'\mathbf{q}_2}\frac{\mathbf{p}_3'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_3'\mathbf{q}_3}{\mathbf{p}_0'\mathbf{q}_3}\text{rest}}}$$

$$\text{rest} = \frac{\mathbf{p}_3'\mathbf{q}_2}{\mathbf{p}_2'\mathbf{q}_2}\frac{\mathbf{p}_3'\mathbf{q}_3}{\mathbf{p}_2'\mathbf{q}_3}\frac{\mathbf{p}_4'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_4'\mathbf{q}_4}{\mathbf{p}_0'\mathbf{q}_4}\frac{\mathbf{p}_2'\mathbf{q}_4}{\mathbf{p}_4'\mathbf{q}_4}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_4'\mathbf{q}_2}\frac{\mathbf{p}_5'\mathbf{q}_0}{\mathbf{p}_0'\mathbf{q}_0}\frac{\mathbf{p}_5'\mathbf{q}_5}{\mathbf{p}_0'\mathbf{q}_5}\frac{\mathbf{p}_2'\mathbf{q}_5}{\mathbf{p}_5'\mathbf{q}_5}\frac{\mathbf{p}_2'\mathbf{q}_2}{\mathbf{p}_5'\mathbf{q}_2}$$

2m-3 indices, 2(2m-3) ratios in the case of Fisher Indices; that is if m = 15 we have 27 Indices and 54 ratios of two aggregates each

9.2 (3) Continuation of the (G)EKS index with the passage of time

2) There is no longer the simplicity of chaining ⟵ the price we have to pay for transitivity

$$P^{EKS}_{01(2)} \rightarrow P^{EKS}_{02(3)} \rightarrow P^{EKS}_{03(4)} \rightarrow P^{EKS}_{04(5)} \rightarrow P^{EKS}_{05(6)}$$

chain Fisher	GEKS
$\overline{P}^F_{02} = \sqrt{P^F_{01}P^F_{12}}$	$P^{EKS}_{02(3)} = \sqrt[3]{\left(P^F_{02}\right)^2 P^F_{01}P^F_{12}}$
$\overline{P}^F_{03} = \sqrt{\overline{P}^F_{02}P^F_{23}}$	$P^{EKS}_{03(4)} = \sqrt[4]{\left(P^{EKS}_{02(3)}\right)^3 \cdot \left(P^F_{03}\right)^2 \cdot \frac{P^F_{13}P^F_{23}}{P^F_{01}P^F_{02}}}$
$\overline{P}^F_{04} = \sqrt{\overline{P}^F_{03}P^F_{34}}$	$P^{EKS}_{04(5)} = \sqrt[5]{\left(P^{EKS}_{03(4)}\right)^4 \cdot \left(P^F_{04}\right)^2 \cdot \frac{P^F_{14}P^F_{24}P^F_{34}}{P^F_{03}P^F_{13}P^F_{23}}}$

To make P^{EKS} independent of the number m of periods to be compared ⇒ RWGEKS

3) RWGEKS another concept of the drift-free target index

RWGEKS with 3 periods window	**GEKS** with m = 6
$P_{02(RW)}^{EKS} = \sqrt[3]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{12}^{F}}$	$P_{02(6)}^{EKS} = \sqrt[6]{\left(P_{02}^{F}\right)^2 P_{01}^{F} P_{13}^{F} P_{03}^{F} P_{32}^{F} P_{04}^{F} P_{42}^{F} P_{05}^{F} P_{52}^{F}}$
$P_{13(RW)}^{EKS} = \sqrt[3]{\left(P_{13}^{F}\right)^2 P_{12}^{F} P_{23}^{F}}$	$P_{03(6)}^{EKS} = \sqrt[6]{\left(P_{03}^{F}\right)^2 P_{01}^{F} P_{13}^{F} P_{02}^{F} P_{23}^{F} P_{04}^{F} P_{43}^{F} P_{05}^{F} P_{53}^{F}}$
$P_{24(RW)}^{EKS} = \sqrt[3]{\left(P_{24}^{F}\right)^2 P_{23}^{F} P_{34}^{F}}$	$P_{04(6)}^{EKS} = \sqrt[6]{\left(P_{04}^{F}\right)^2 P_{01}^{F} P_{14}^{F} P_{02}^{F} P_{24}^{F} P_{03}^{F} P_{34}^{F} P_{05}^{F} P_{54}^{F}}$
$P_{35(RW)}^{EKS} = \sqrt[3]{\left(P_{35}^{F}\right)^2 P_{34}^{F} P_{45}^{F}}$	$P_{05(6)}^{EKS} = \sqrt[6]{\left(P_{05}^{F}\right)^2 P_{01}^{F} P_{15}^{F} P_{02}^{F} P_{25}^{F} P_{03}^{F} P_{35}^{F} P_{04}^{F} P_{45}^{F}}$

Is rolling window GEKS (= RWGEKS) P_{24}, P_{35}, … really comparable to P_{04} , P_{05}, … or doesn't it need to be chained?

9.2 (5) Rolling window RWGEKS and chaining

4) RWGEKS index a link rather than a chain

The series of RWGEKS indices cannot possibly be viewed as comparable to P_{03}, P_{04}, … because they cover only a part of the tie series. If we take them as links and and take $P^{EKS}_{02(RW)}$ as starting point, we get

chained RWGEKS indices	GEKS index (full interval 0 – 6)
$P^{EKS}_{02(RW)} = \sqrt[3]{\left(P^F_{02}\right)^2 P^F_{01} P^F_{12}}$	$\sqrt[6]{\left(P^F_{02}\right)^2 P^F_{01} P^F_{13} P^F_{03} P^F_{32} P^F_{04} P^F_{42} P^F_{05} P^F_{52}}$
$\overline{P}^{EKS}_{03(RW)} = P^{EKS}_{02(RW)} P^{EKS}_{13(RW)}$ $= \sqrt[3]{P^F_{01}\left(P^F_{02}\right)^2\left(P^F_{13}\right)^2\left(P^F_{12}\right)^2 P^F_{23}}$	$\sqrt[6]{\left(P^F_{03}\right)^2 P^F_{01} P^F_{13} P^F_{02} P^F_{23} P^F_{04} P^F_{43} P^F_{05} P^F_{53}}$
$\overline{P}^{EKS}_{04(RW)} = P^{EKS}_{02(RW)} P^{EKS}_{13(RW)} P^{EKS}_{24(RW)}$ $= \sqrt[3]{P^F_{01}\left(P^F_{02}\right)^2\left(P^F_{13}\right)^2\left(P^F_{12}\right)^2\left(P^F_{23}\right)^2\left(P^F_{24}\right)^2 P^F_{34}}$	$\sqrt[6]{\left(P^F_{04}\right)^2 P^F_{01} P^F_{14} P^F_{02} P^F_{24} P^F_{03} P^F_{34} P^F_{05} P^F_{54}}$

↑ no chain drift as well??

↑ no chain drift

Relation between successive RWGEKS indices is simple

$$P_{02,RW}^{EKS} \to P_{13,RW}^{EKS} \to P_{24,RW}^{EKS} \to P_{24,RW}^{EKS}$$

$$P_{13(RW)}^{EKS} = P_{02(RW)}^{EKS} \sqrt[3]{\left(P_{13}^{F}\right)^2 P_{23}^{F} \Big/ \left(P_{02}^{F}\right)^2 P_{01}^{F}}$$

$$P_{24(RW)}^{EKS} = P_{13(RW)}^{EKS} \sqrt[3]{\left(P_{24}^{F}\right)^2 P_{34}^{F} \Big/ \left(P_{13}^{F}\right)^2 P_{12}^{F}}$$

However RWGEKS fails time reversal test and multiperiod identity?

assume same prices in 0 and 4 then $P_{04}^{F} = 1$

and also $P_{04(6)}^{EKS} = \sqrt[6]{P_{01}^{F} P_{14}^{F} P_{02}^{F} P_{24}^{F} P_{03}^{F} P_{34}^{F} P_{05}^{F} P_{54}^{F}} = 1$

however, RWGEKS (3 periods window) index $\neq$ **1** even if prices in 2 and 4 are the same

$$P_{24(RW)}^{EKS} = \sqrt[3]{\left(P_{24}^{F}\right)^2 P_{23}^{F} P_{34}^{F}}$$

↓

we then get $P_{24(RW)}^{EKS} = \sqrt[6]{\frac{\sum p_3 q_2}{\sum p_2 q_2} \frac{\sum p_2 q_4}{\sum p_3 q_4}} \neq 1$ unless also $q_{2i} = q_{4i}$

GEKS and RWGEKS cannot both be the "drift-free" target

RWGEKS differs from direct Fisher

a system is hard to discover here

$$P^{EKS}_{02(RW)} / P^F_{02} = \sqrt[3]{P^F_{01} P^F_{12} P^F_{20}}$$

$$P^{EKS}_{13(RW)} / P^F_{03} = \sqrt[3]{P^F_{12} \left(P^F_{13}\right)^2 P^F_{23} \left(P^F_{30}\right)^3} \qquad P^{EKS}_{13(RW)} / P^F_{13} = \sqrt[3]{P^F_{12} P^F_{23} P^F_{31}}$$

and also from GEKS

$$P^{EKS}_{02(RW)} / P^{EKS}_{02(6)} = \sqrt[6]{P^F_{01} \left(P^F_{02} P^F_{12}\right)^2 P^F_{23} P^F_{24} P^F_{25} P^F_{30} P^F_{31} P^F_{40} P^F_{50}}$$

$$P^{EKS}_{13(RW)} / P^{EKS}_{03(6)} = \sqrt[6]{P^F_{10} \left(P^F_{12}\right)^2 \left(P^F_{13}\right)^3 P^F_{20} P^F_{23} \left(P^F_{30}\right)^2 P^F_{34} P^F_{35} P^F_{40} P^F_{50}}$$

$$P^{EKS}_{24(RW)} / P^{EKS}_{04(6)} = \sqrt[6]{P^F_{10} P^F_{20} \left(P^F_{23}\right)^2 \left(P^F_{24}\right)^3 P^F_{30} P^F_{34} \left(P^F_{40}\right)^2 P^F_{41} P^F_{45} P^F_{50}}$$

on the RHS in each case 14 indices

Now which is the correct (target) index and consequently which is the correct drift?

9.2 (8) Summary of the critique to GEKS and RWGEKS indices of IFD*

* I sent this critique and all slides of the part IV (January 2010) to Erwin Diewert and he agreed with it. He sees, however, some great advantages of the method from a practical point of view.

1. (G)EKS method complicated, indices are depending on the number of periods taken into consideration, and they are difficult to interpret
2. RWGEKS not a solution:
   - another concept of "drift-free",
   - need to chain-link RWGEKS indices (the chained RWGEKS differ from the GEKS indices and therefore will no longer be drift-free, and
   - they fail the time reversal test and multiperiod-identity test
3. no simple chain-linking formula for GEKS (as opposed to RWGEKS) exists when new observations appear and index needs to be continued

9.3 (1) Pseudo-Fisher Indices when baskets are updated infrequently only

DHK = W. E. Diewert, M. Huwiler, U. Kohli, Retrospective Price Indices and Substitution Bias, Oct. 2008

1. What can be done with **direct indices** (Paasche, Fisher) when baskets can only be up-dated after T periods?* 0, ..., t, ..., T
2. Problem interesting because **chain-indices** are justified by the need for a timely and speedy up-dating of weights, and now we see that this might not be possible in practice
3. We don't show which considerations lead to the "**Pseudo Fisher Index"** (PFI) idea as "retrospective measure of the price level"

$$P_{01}^{PFI} = \sqrt{K \sum p_{i1} q_{i0} \sum p_{i1} q_{iT}} \quad \text{where} \quad K = \left(\sum p_{i0} q_{i0} \sum p_{i0} q_{iT} \right)^{-1}$$

general: $$P_{0t}^{PFI} = \sqrt{K \sum p_{it} q_{i0} \sum p_{it} q_{iT}}$$

* e.g. every T = 5 years only

9.3 (2) Pseudo-Fisher Indices when baskets are updated infrequently only

the last term (t = T) of the sequence is equal to the direct Fisher Index

$$P_{0T}^{PFI} = \sqrt{K \sum p_{iT} q_{i0} \sum p_{iT} q_{iT}} = \sqrt{\frac{\sum p_{iT} q_{i0} \sum p_{iT} q_{iT}}{\sum p_{i0} q_{i0} \sum p_{i0} q_{iT}}}$$

for the intermediate terms we get

Fisher (F)	Pseudo-Fisher (PFI)	squared drift $(P^{PFI}/P^{F})^2$
$P_{01}^{F} = \sqrt{\frac{\sum p_{i1} q_{i0} \sum p_{i1} q_{i1}}{\sum p_{i0} q_{i0} \sum p_{i0} q_{i1}}}$	$P_{01}^{PFI} = \sqrt{\frac{\sum p_{i1} q_{i0} \sum p_{i1} q_{iT}}{\sum p_{i0} q_{i0} \sum p_{i0} q_{iT}}}$	$P_{01(T)}^{LO} / P_{01}^{P}$
$P_{02}^{F} = \sqrt{\frac{\sum p_{i2} q_{i0} \sum p_{i2} q_{i2}}{\sum p_{i0} q_{i0} \sum p_{i0} q_{i2}}}$	$P_{02}^{PFI} = \sqrt{\frac{\sum p_{i2} q_{i0} \sum p_{i2} q_{iT}}{\sum p_{i0} q_{i0} \sum p_{i0} q_{iT}}}$	$P_{02(T)}^{LO} / P_{02}^{P}$

numerator is a Lowe price index

first factor is P^L

Lowe index weights of T $P_{0t(T)}^{LO} = \frac{\sum p_{it} q_{iT}}{\sum p_{i0} q_{iT}}$

9.3 (3) Pseudo-Fisher Indices (PFI) of DHK: interpretation

The squared drift $(P^{PFI}/P^{F})^2$ can be described as ratio of two price indices $(p_{i0} \to p_{it})$ P^{LO}/P^{P} or as ratio of two quantity indices $(q_{it} \to q_{iT})$ Q^{P}/Q_{LO}

$$\left(\frac{P_{0t}^{PFI}}{P_{0t}^{F}}\right)^2 = \frac{\sum p_{it}q_{iT}}{\sum p_{i0}q_{iT}} : \frac{\sum p_{it}q_{it}}{\sum p_{i0}q_{it}} = \frac{\sum p_{it}q_{iT}}{\sum p_{it}q_{it}} : \frac{\sum p_{i0}q_{iT}}{\sum p_{i0}q_{it}}$$

$$P_{0t(T)}^{LO} = \frac{\sum p_{it}q_{iT}}{\sum p_{i0}q_{iT}} = \frac{P_{Tt}^{L}}{P_{T0}^{L}} = P_{0T}^{P}P_{Tt}^{L}$$

$$P_{0t}^{P}$$

$$Q_{tT}^{L}$$

$$Q_{tT(0)}^{LO} = \frac{\sum p_{i0}q_{iT}}{\sum p_{i0}q_{iz}} = \frac{Q_{0T}^{L}}{Q_{0t}^{L}} = Q_{0T}^{L}Q_{t0}^{P}$$

$$\left(\frac{P_{0t}^{PFI}}{P_{0t}^{F}}\right)^2 = \frac{P_{0t(T)}^{LO}}{P_{0t}^{P}} = \frac{\sum p_{it}q_{iT}}{\sum p_{i0}q_{iT}} : P_{0t}^{P} = \frac{A}{B} : P_{0t}^{P}$$

$$\left(\frac{P_{0t}^{PFI}}{P_{0t}^{L}}\right)^2 = \frac{P_{0t(T)}^{LO}}{P_{0t}^{L}} = \frac{A}{B} : P_{0t}^{L}$$

because of P^{LO} the theorem of L. v. Bortkiewicz does not apply to the squared bias P^{PFI}/P^{L} and P^{PFI}/P^{F}

9.3 (4) Pseudo-Fisher Indices PFI and other indices; sequences of PFIs

PFI and mid-year (= Marshall Edgeworth ME) index

$$\frac{\sum p_{it}(q_{i0}+q_{iT})}{\sum p_{i0}(q_{i0}+q_{iT})}=\frac{P_{0t}^{L}+A}{1+B}$$

by contrast

$$P_{0t}^{PFI}=\sqrt{P_{0t}^{L}\frac{\sum p_{it}q_{iT}}{\sum p_{i0}q_{iT}}}=\sqrt{P_{0t}^{L}\frac{A}{B}}$$

successive Pseudo Fisher indices (PFIs) and chaining

$$\frac{P_{0t}^{PFI}}{P_{0,t-1}^{PFI}}=\sqrt{\frac{\sum p_{it}q_{i0}}{\sum p_{i,t-1}q_{i0}}\frac{\sum p_{it}q_{iT}}{\sum p_{i,t-1}q_{iT}}}=\sqrt{P_{t(0)}^{LO}P_{t(T)}^{LO}}$$

geometric mean of Lowe links

successive direct Fisher indices and chaining

Formula for successive values of P^F (direct Fisher) is complicated. It is not a geometric mean of a Laspeyres and Paasche link

$$\frac{P_{0t}^{F}}{P_{0,t-1}^{F}}=\sqrt{P_t^{L}P_t^{P}P_{t(0)}^{LO}\frac{\sum p_{t-1}q_t}{\sum p_0 q_t}\frac{\sum p_0 q_{t-1}}{\sum p_t q_{t-1}}}\neq\sqrt{P_t^{L}P_{t)}^{P}}$$

Just like the direct Fisher Index the PFI has no mean-of-relatives nor a ratio-of-expenditures interpretation. DHK give an interpretation as geometric mean of price indices using expenditure shares s_{i0} and $s_{iT} = p_{iT}q_{iT}/\Sigma p_{iT}q_{iT}$

$$\left(P_{0t}^{PFI}\right)^2 = \sum s_{i0} \frac{p_{it}}{p_{i0}} \frac{\sum s_{iT} \frac{p_{it}}{p_{iT}}}{\sum s_{iT} \frac{p_{i0}}{p_{iT}}} = P_{0t}^{L} \frac{P_{0T}^{P}}{P_{tT}^{P}}$$

With $C = \Sigma p_T q_T$ this is equal to

$$\left(P_{0t}^{PFI}\right)^2 = P_{0t}^{L} \frac{A/C}{B/C} = P_{0t}^{L} \frac{A}{B}$$

1. Method seems plausible when q_{it} not available
2. however, interpretation of P^{PFI} formula and its drift in terms of Laspeyres and Lowe indices not simple

   (Pices 0 and t, quantities T)
3. for successive P^{PFI} indices exists a simple chain-linking formula (by contrast to PF)

9.4 (1) Optimal intervals for linking (IFL) of chain indices (chaining with variable IFL)

When the interval (0,s) over which an index P_{0s} is defined is shorter than the period under consideration (0,t) chain-linking is required. However, this can be done over **intervals for linking (IFL**s) of different length. In general a **uniform** IFL of **one** period is assumed, however, it can also be 2 or 3 periods, and the **IFLs can also vary** from sub-interval to sub-interval. It is well known that the results will differ depending on length and spacing:

good	p_0	q_0	p_1	q_1	p_2	q_2	p_3	q_3	p_4	q_0
1	1	3	2	1.4	1.5	2.2	1	3	1.5	2.5
2	1	2	0.3	5	0.5	1.7	1	2	0.6	1.8

Now consider some examples with IFLs of different length and spacing

0	1	2 - 3	4

$$\overline{P}_{04}^{L(1)} = P_{01}^{L} P_{13}^{L} P_{34}^{L} = 2.2397$$

$P_{13} \approx 1.5$
$P_{12}P_{23} \approx 1$

$$\overline{P}_{04}^{L} = P_{01}^{L} P_{12}^{L} P_{23}^{L} P_{34}^{L} = 1.5128$$

0 - 2	3	4

$$\overline{P}_{04}^{L(2)} = P_{02}^{L} P_{23}^{L} P_{34}^{L} = 1.1785$$

0- 3	4

$$\overline{P}_{04}^{L(3)} = P_{03}^{L} P_{34}^{L} = 1 \cdot (5.7/5) = 1{,}14$$

0 - 4

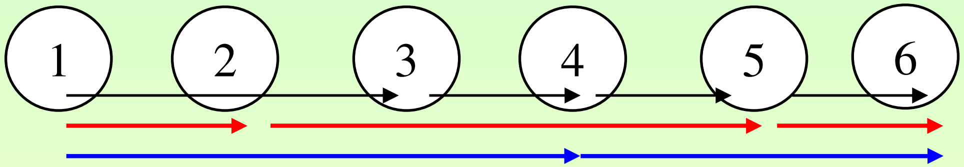
$$\overline{P}_{04}^{L(4)} = P_{04}^{L} = 5.7/5 = 1.14$$

With IFLs of variable length **the distinction between direct and chain indices becomes blurred**. The latter is simply the special case of only one link and a length of t periods

The example for periods 0 to 3 is taken from **Ehemann**. He found a chained Törnquist index violating identity: $\overline{P}_{02}^{T} = 0{,}92$ and $\overline{P}_{03}^{T} = 0.85 \neq 1$

9.4 (2) Optimal intervals for linking (IFL) of chain indices

Ehemann 2005 suggests: To avoid chain drift linking should be done between periods which are most alike and not necessarily between periods adjacent in time. Similarity of relative prices* is "the criterion for selecting periods as endpoints for linking". If there are observations (e.g. in period 1 and 2) in which prices change dramatically** they may be skipped, and one could chain directly from 0 to 3. To find "optimal" IFLs amounts to selecting "the shortest path through a graph from P_0 to P_t"



three examples of possible linkings → → and →

Some final conclusions of C. Ehemann:

- Lengthening the linking interval has less effect on Tornqvist rather than Fisher index
- Choosing the optimal linking period for each particular aggregate and period can yield widely different lengths (one quarter to several years)
- "When index numbers are calculated with different linking intervals, the rate of change in the aggregate is ambiguous due to chain drift."
- Estimates of recent periods must be regarded as provisional. For more observations →

* measured as weighted difference between $\ln(p_{it})$ and $\ln(p_{i,t-1})$ ** for example a non-recurring spike in one period

9.4 (3) Optimal intervals for linking (IFL) of chain indices

The method

1. no longer uses (arbitrary) a priori defined uniform IFLs for all indices (aggregates) and all periods, and "observations in which there are important temporary price changes will be skipped"
2. cannot be applied in real time, only retrospectively for time series of some length
3. needs a **concept of similarity** of price structures (see more in **sec. 9.5**, on next slides). Ehemann made use of a distance λ_{jk} between prices of any two (j, k) periods (simply the log of the ratio of two Tornqvist indices) and a quadratic loss function L using weighted squares of all $\ln p_{ij} - \ln p_{ik}$ from all possible λ_{jk} (periods j, and k with small λ_{jk} are chosen as endpoints of links; other distance and loss functions may lead to different results)
4. may also be viewed as to seek the shortest path through a directed graph*(involving quite complicated algorithms)
5. has been given a economic theory interpretation (link points in a chain are viewed as equilibrium (utility maximization) points (however, significant differences to official statistics occurred mainly in economically most turbulent periods when the assumptions of this approach are unlikely to prevail.

* The minimization method used here differs, however, from Hill's ***minimum spanning tree***, used in the context of international comparisons (described in v.d.Lippe (2007), pp. 525 - 529)

9.5 (1) A theory of dissimilarity indices (Diewert)

Diewert (Febr. 2010) took an axiomatic approach to indices of dissimilarity of price and quantity vectors (application: regional linking, outlier detection, "deciding how to aggregate up a large number of price and quantity series into smaller number of aggregates")

absolute dissim. (D) of prices* $p_{1i} \neq p_{2i}$ (good i) or quantities. In general $x_i \neq y_i$	**relative** dissimilarity (Δ) $p_{1i} \neq \lambda p_{2i}$ ($\lambda > 0$) (structure, not level of prices)

Axioms: (Axioms A refer to the N = 1 variable case (instead of vectors **x** and **y**)

B1 Continuity Dd(x,y) continuous function for all x > 0, y > 0	**C1** Continuity of Δ(x,y)
B2 Identity D(x,x) = 0	**C2** Identity
B3 Positivity (D is a positive scalar)	**C3** Positivity
B4 Symmetry D(x,y) = D(y,x)	**C4** Symmetry
B5 Invariance to changes in units of measurement	**C5** Invariance to changes in units of measurement
B6 Monotonicity D(x,y) is increasing in y if y ≥ x **	**C6** Invariance to the ordering of commodities (there is no counterpart to A6)
	C7 Proportionality Δ(x, λy) =Δ(x.y)

* or vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ ** additional axioms to reduce the class of admissible functions: **B7** = invariance to the ordering of commodities, **B8** = additive separability

Absolute dissimilarity index functions D(x,y) satisfying B1 – B8

asymptotically linear index: $D_{AL}(x,y) = (1/N)\Sigma_i[(y_i/x_i) -1 + (x_i/y_i) - 1]$
as. quadratic index: $D_{AQ}(x,y) = (1/N)\Sigma_i[(y_i/x_i) -1]^2 + (1/N)\Sigma_i[(y_i/x_i) -1]^2$
log squared (Jevons) index $D_{LS}(x,y) = (1/N)\Sigma_i[\ln(yi/xi)]^2$

Replacing B8 by the weaker axiom of componentwise symmetry allows to obtain an axiomatic characterization (uniqueness theorem)

To arrive at an **index of relative dissimilarity** Δ(x,y) Diewert proposed

1. to find a scale index S(**x**,**y**)* to scale up the vector **x** in order to make it comparable to **y**, and
2. to use S(**x**,**y**)x instead of x in D(x,y) so that Δ(x,y) = D(S(x,y)x,y)

Using the Cauchy Schwarz inequality $0 < C = (\mathbf{x}'\mathbf{y})^2/(\mathbf{x}'\mathbf{x})(\mathbf{y}'\mathbf{y}) \leq 1$ to form Δ = 1- C indices Diewert found indices as symmetric means of C terms where vectors **x**, **y** are substituted as follows: **x** = **r** (where $r_i = y_i/x_i$), $\mathbf{y} = \mathbf{1}_N$ and x = s (where $s_i = x_i/y_i$) and $\mathbf{y} = \mathbf{1}_N$ respectively

* essentially a price or quantity index

10.1 Collateral damages of chain indices: axioms no longer relevant

According to Dr. R.* the "mean value property" is nothing a meaningful index should necessarily possess, because we have chain indices (required even by law) and they also violate additivity. So as chain indices violate some axioms these axioms can no longer be regarded as reasonable or even desirable.

This is clearly erroneous:

1. mean value property and additivity (of volumes) should be kept distinct
   for example deflation using direct Fisher indices as deflators will result in non-additive volumes, however, the direct Fisher index clearly fulfils the mean value property
2. non-additivity applies only to the chain, not to the link, and it is the latter on which the focus lies in practice
3. in addition to the mean value property chain indices violate many more axioms (e.g. identity and monotonicity as demonstrated in part I of the course), so following the logic of Dr. R. practically any nonsense–index (as for example the R-index, next slide) could be justified

* I am here referring to an unpublished, unofficial paper, therefore no name quoted

10.1 Collateral damages (2)

consider the R-Index

$$P_{0t}^{R} = \frac{\sum \left(p_{it}^{2} - p_{i0}\right) q_{i0}}{\sum \left(p_{it}^{2} + p_{i0}\right) q_{i0}}$$

instead of the L (Laspeyres) index

Identity

p_{i0}	p_{it}	q_{i0}
3	3	3
4	4	2
5	5	1

R-Index 0.5849

L index 1

Monotonicity

p_{i0}	p_{it}	q_{i0}
3	3	3
4	4	2
5	6	1

The index should clearly rise (5 → 6), however

R-Index 0.6239

L index 1.045

Mean value property

p_{i0}	p_{it}	q_{i0}	relative
3	4.5	3	1.5
4	5	2	1.25
5	4	1	0.8

R-Index 0.7042

L index 1.25

However, according to Dr. R the L-index is by no means better than the R-index

Appendix 1 Personal note

I found **quotations of my book "Chain Indices"** (if it is quoted at all) in which reference is made to an argument of mine, which I think is of not much importance and relevance (I was not even aware of having said that) . It reads as follows

"… there has been the objection to chain indices because they have no counterpart in the spatial context. In a time-series context, there is a natural ordering of the sequence of chaining from t to t+1 to t+2 etc. In a spatial comparison, between countries or regions no such ordering exists and, for reasons of consistency, fixed-base indices should be used in both temporal and spatial comparisons (von der Lippe 2001)."*

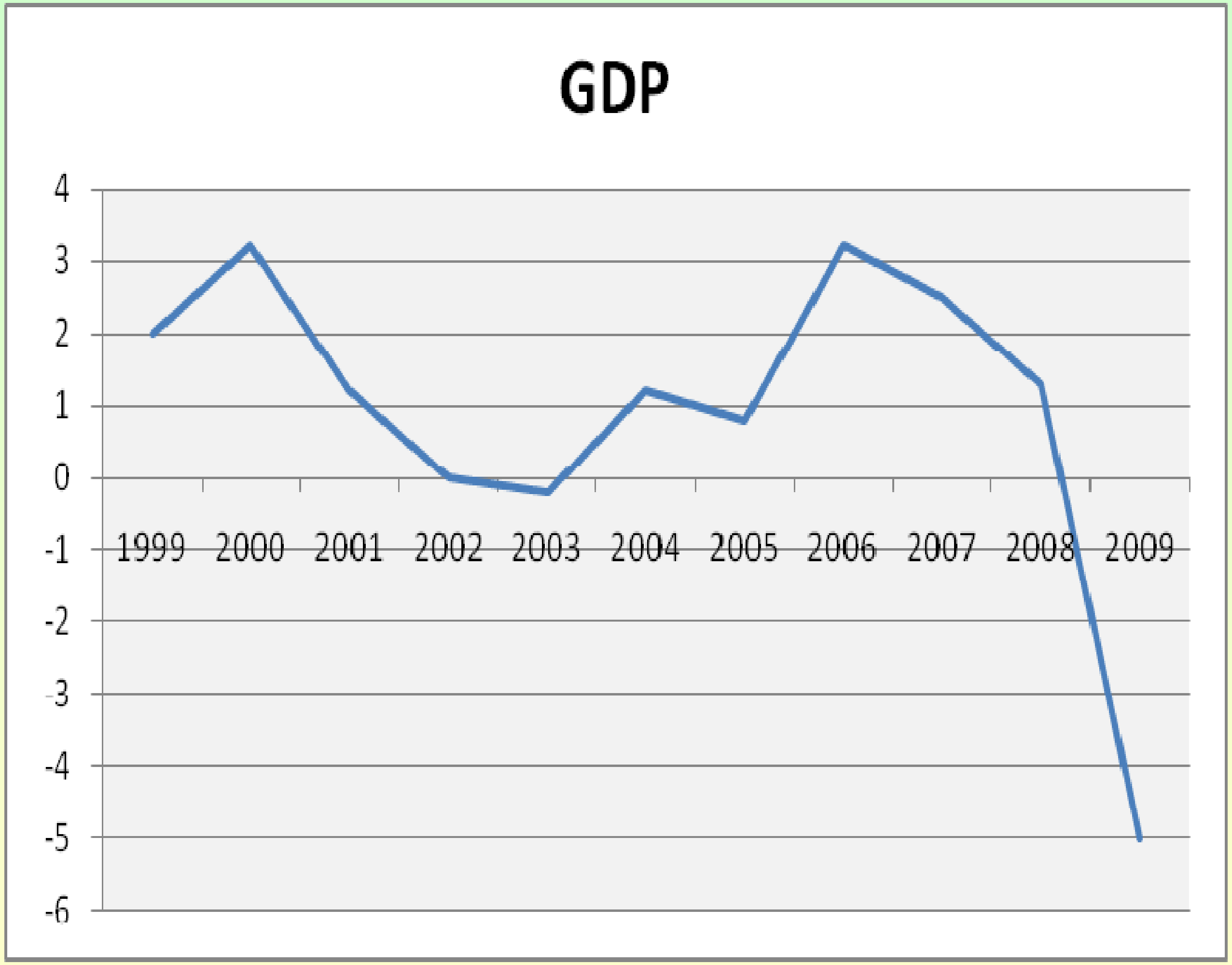
* Paul Schreyer, Chain Index Number Formulae in the National Accounts, 8th OECD-NBS Workshop on National Accounts, Paris, 6-10 Dec. 2004. Also Diewert (§90 in his chapter "Basic Index Number Theory" of the ICP Manual) quoted me with the same (in my view rather weak) argument.

I did not find, however, quotations of my book "Chain Indices", in which comments are made on my arguments against chain indices or – more important - my attempts to refute arguments of "chainers" in favour of chain indices (chapter 6). These, however, are the parts of the book, I consider the most important ones. They are quoted in particular in part I (slides 25 - 40) of this presentation.

Appendix 2 Annual growth rates GDP chained, volume DE

The downswing in 2009 due to the "financial crisis" is clearly visible

year	GDP
1999	2,0
2000	3,2
2001	1,2
2002	0
2003	- 0,2
2004	1,2
2005	0,8
2006	3,2
2007	2,5
2008	1,3
2009	- 5,0



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